

HOMEWORK 4 FOR 18.125, SPRING 2010
DUE THURSDAY, MARCH 4 AT THE BEGINNING OF
LECTURE.

HW4.1 Stroock 3.2.16. In addition, let f be a non-negative measurable function on a measure space $(E, \mathcal{B}, \mu)$. Show that $\int_E f d\mu = 0$ implies $\mu(f \neq 0) = 0$.

HW4.2 Stroock 3.2.17.

HW4.3 Suppose f is a non-negative integrable function on a measure space $(E, \mathcal{B}, \mu)$ such that. For any $\Gamma \in \mathcal{B}$ set

$$\nu(\Gamma) = \int_{\Gamma} f d\mu.$$

Show ν is a measure on E with the same σ -algebra $\mathcal{B}$. Does the same conclusion hold if $\int_E f d\mu = \infty$?

HW4.4 Let $f : [0, 1] \rightarrow \mathbb{R}$ be a bounded, non-negative, Riemann integrable function. Prove that f is measurable with respect to Lebesgue measure, and that

$$\int_0^1 f dx = (R) \int_0^1 f dx,$$

where the left hand integral is Lebesgue integration. (*Hint:* Approximate f above and below by sequences of simple functions.)

HW4.5 Let f be a non-negative integrable function on a measure space $(E, \mathcal{B}, \mu)$. Show that for each $\epsilon > 0$, there exists $\delta > 0$ such that $\Gamma \in \mathcal{B}$, $\mu(\Gamma) < \delta$ implies $\int_{\Gamma} f d\mu < \epsilon$.