HOMEWORK 2 FOR 18.125, SPRING 2010
DUE THURSDAY, FEBRUARY 18 AT THE BEGINNING OF LECTURE.

HW2.1 Show that any countable subset of $\mathbb{R}^N$ is measurable and has Lebesgue measure zero. Prove that any countable union of sets of Lebesgue measure zero has Lebesgue measure zero. Let $A \subset \mathbb{R}$ be defined by

$$A = [0, 1] \setminus \mathbb{Q}.$$

Prove A is measureable and determine the measure.

HW2.2 Stroock 2.1.19 (it may be helpful to look at 2.1.18).

HW2.3 Stroock 2.1.20.

HW2.4 Prove that for a compact set $A \subset \mathbb{R}^N$, Lebesgue measure can be computed using only finite coverings:

$$|A| = \inf \{\Sigma(\mathcal{C}) : \mathcal{C} \text{ is a finite cover of } A \text{ by rectangles}\}.$$

HW2.5 Let $\{\Gamma_n\}_{n=1}^{\infty}$ be a sequence of measurable sets such that $\Gamma_n \cap \Gamma_m = \emptyset$ if $n \neq m$. Let $A \subset \mathbb{R}^N$ be any set, and prove

$$|A \cap (\cup_{n=1}^{\infty} \Gamma_n)|_e = \sum_{n=1}^{\infty} |A \cap \Gamma_n|_e.$$