

HOMEWORK 1 FOR 18.125, SPRING 2010
DUE THURSDAY, FEBRUARY 11 AT THE BEGINNING OF
LECTURE.

HW1.1 Let $J = [a, b] \subset \mathbb{R}$ and let $a_0, \dots, a_n$ satisfy $a = a_0 < a_1 < \dots < a_n = b$. Assume $\psi : J \rightarrow \mathbb{R}$ is constant on each interval (a_m, a_{m+1}) , $0 \leq m \leq n-1$. Show that if $\phi \in C(J)$, then ϕ is ψ -Riemann integrable and

$$(R) \int_J \phi(x) d\psi(x) = \sum_{m=0}^n \phi(a_m) d_m,$$

where the numbers d_m are given by

$$d_0 = \psi(a+) - \psi(a), \quad d_m = \psi(a_{m+1}) - \psi(a_m), \quad \text{for } 1 \leq m \leq n-1, \quad d_n = \psi(b) - \psi(b-).$$

HW1.2 Stroock 1.1.9

HW1.3 Stroock 1.2.26

HW1.4 Suppose $\psi : J \rightarrow \mathbb{R}$ is non-decreasing and $f : J \rightarrow \mathbb{R}$ is ψ -Riemann integrable. Assume $m \leq f \leq M$ on J and $\phi : [m, M] \rightarrow \mathbb{R}$ is a continuous function. Prove $\phi \circ f : J \rightarrow \mathbb{R}$ is ψ -Riemann integrable on J .

HW1.5 Let $\psi : J \rightarrow \mathbb{R}$ be a non-decreasing function. Let $p, q \in \mathbb{R}$ be positive numbers satisfying

$$\frac{1}{p} + \frac{1}{q} = 1.$$

Prove:

(a) If $u, v \geq 0$, then

$$uv \leq \frac{u^p}{p} + \frac{v^q}{q}.$$

(b) If $f, g \geq 0$ are ψ -Riemann integrable functions on J , then

$$\left| \int_J fg d\psi \right| \leq \left(\int_J f^p d\psi \right)^{1/p} \left(\int_J g^q d\psi \right)^{1/q}.$$

(Hint: If both integrals on the right hand side are non-zero, renormalize the problem so they are both equal to 1 and use part (a).)