Second homework assignment in 18.101

1. Munkres §10, # 5

2. Munkres §10, # 6

3. Let $U$ be an open subset of $\mathbb{R}^2$ and $f : U \to \mathbb{R}$ a $C^2$-differentiable function. Use Fubini’s theorem to prove that

$$\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial^2 f}{\partial y \partial x}(x,y)$$

4. Munkres §12, # 2

5. Munkres §12, # 3. Hint: Here are some hints about how to construct a subset, $S$, of $Q$ with the properties described in the “Hint” in part c.

  a. A point, $q \in Q$ is a rational point if all its coordinates are rational numbers. Show that the set of rational points in $Q$ is countable.

  b. Let $q_1, q_2, q_3, \ldots$ be an enumeration of the set of rational points in $Q$. Show by induction that there exists a set of points, $p_1, p_2, p_3, \ldots$ in $Q$ such that $|p_N - q_N| < \frac{1}{N}$ and such that the coordinates of $p_N$ are distinct from the coordinates of $p_i$ for $i < N$.

  c. Let $S$ be the set of $p_i$'s.