Second homework assignment in 18.101

1. Munkres § 10, # 5
2. Munkres § 10, # 6
3. Let U be an open subset of $\mathbb{R}^2$ and $f : U \longrightarrow \mathbb{R}$ a C^2-differentiable function. Use Fubini's theorem to prove that

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{\partial^2 f}{\partial y \partial x}(x, y)$$

4. Munkres § 12, # 2
5. Munkres § 12, # 3. *Hint*: Here are some hints about how to construct a subset, S, of Q with the properties described in the "Hint" in part c.
   a. A point, $q \in Q$ is a *rational* point if all its coordinates are rational numbers. Show that the set of rational points in Q is countable.
   b. Let $q_1, q_2, q_3, \ldots$ be an enumeration of the set of rational points in Q. Show by induction that there exists a set of points, $p_1, p_2, p_3, \ldots$ in Q such that $|p_N - q_N| < \frac{1}{N}$ and such that the coordinates of p_N are distinct from the coordinates of p_i for $i < N$.
   c. Let S be the set of p_i's.