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Second homework assignment in 18.101  
due Monday, October 6

1. Munkres §10, # 5
2. Munkres §10, # 6
3. Let  $U$  be an open subset of  $\mathbb{R}^2$  and  $f : U \rightarrow \mathbb{R}$  a  $C^2$ -differentiable function. Use Fubini's theorem to prove that

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{\partial^2 f}{\partial y \partial x}(x, y)$$

4. Munkres §12, # 2
5. Munkres §12, # 3. *Hint*: Here are some hints about how to construct a subset,  $S$ , of  $Q$  with the properties described in the "Hint" in part c.
  - a. A point,  $q \in Q$  is a *rational* point if all its coordinates are rational numbers. Show that the set of rational points in  $Q$  is countable.
  - b. Let  $q_1, q_2, q_3, \dots$  be an enumeration of the set of rational points in  $Q$ . Show by induction that there exists a set of points,  $p_1, p_2, p_3, \dots$  in  $Q$  such that  $|p_N - q_N| < \frac{1}{N}$  and such that the coordinates of  $p_N$  are distinct from the coordinates of  $p_i$  for  $i < N$ .
  - c. Let  $S$  be the set of  $p_i$ 's.