

Second homework assignment in 18.101
due Friday, October 8

1. Munkres §10, # 5
2. Munkres §10, # 6
3. Let U be an open subset of \mathbb{R}^2 and $f : U \rightarrow \mathbb{R}$ a C^2 -differentiable function. Use Fubini's theorem to prove that

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{\partial^2 f}{\partial y \partial x}(x, y)$$

Hint: See Spivak, page 61, # 3.28

4. Munkres §12, # 2
5. Munkres §12, # 3. *Hint:* Here are some hints about how to construct a subset, S , of Q with the properties described in the "Hint" in part c.
 - a. A point, $q \in Q$ is a *rational* point if all its coordinates are rational numbers. Show that the set of rational points in Q is countable.
 - b. Let q_1, q_2, q_3, \dots be an enumeration of the set of rational points in Q . Show by induction that there exists a set of points, p_1, p_2, p_3, \dots in Q such that $|p_N - q_N| < \frac{1}{N}$ and such that the coordinates of p_N are distinct from the coordinates of p_i for $i < N$.
 - c. Let S be the set of p_i 's.