Problem Set 9

Due December 1st at 4 pm in room 2-108.

Hand in parts 1, 2 and 3 separately. Put your name and whether you are registered for 18.100B or 18.100C on each part.

Part 1

1. Problem 10, parts a – c, from page 139. *Hint:* For part a, use the result of Problem 7 from Problem Set 8.
2. Let $x > 0$, let $n \in \mathbb{N} \cup \{0\}$, and let $f\colon [0, x] \to \mathbb{R}$ be $n+1$ times differentiable with $f^{(n+1)}$ integrable. Use mathematical induction and integration by parts to prove that
$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + I_n(x),$$
where
$$I_n(x) = \frac{x^{n+1}}{n!}\int_0^1 (1-t)^n f^{(n+1)}(tx)dt = \frac{1}{n!}\int_0^x (x-x')^n f^{(n+1)}(x')dx'.$$
Theorem 5.15 in Rudin uses the mean value theorem to prove another version of Taylor's theorem under slightly weaker hypotheses, but this version has the advantage of giving a more explicit remainder. Of course the case $x < 0$ follows by applying the result to $g(x) = f(-x)$, and an expansion near $a \neq 0$ follows by taking $g(x) = f(a + x)$.

Part 2

3. Problem 15 from page 141.
4. Problem 2 from page 165.

Part 3

5. Problem 3 from page 165.
6. Let $f\colon [0, 1] \to [0, 1]$ be continuously differentiable with nonincreasing derivative. Prove that the arclength of the graph of f is at most 3.