

Problem Set 7

Due November 3rd at 4 pm in room 2-108.

Hand in parts 1, 2 and 3 separately. Put your name and whether you are registered for 18.100B or 18.100C on each part.

Part 1

1. Let X and Y be metric spaces and $f: X \rightarrow Y$ a function. Prove that f is continuous if and only if $f(\overline{E}) \subset \overline{f(E)}$ for any subset $E \subset X$.
2. Problem 10 from page 99.

Part 2

3. Problem 14 from page 100.
4. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be continuous and suppose that $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ both exist and are finite. Prove that f is uniformly continuous.

Part 3

5. Let K be a compact metric space with metric d and suppose $f: K \rightarrow K$ is *distance preserving*, meaning that $d(f(x), f(y)) = d(x, y)$ for all $x, y \in K$. Prove that $f(K) = K$.
6. Problem 1 from page 114.