

## Problem Set 10

Due December 8th at 4 pm in room 2-108.

Hand in parts 1, 2 and 3 separately. Put your name and whether you are registered for 18.100B or 18.100C on each part.

### Part 1

1. Let  $K: [0, 1] \times [0, 1] \rightarrow \mathbb{R}$  be continuous, and let  $\mathcal{F}$  be the family of functions  $f$  from  $[0, 1]$  to  $\mathbb{R}$  satisfying

$$f(x) = \int_0^1 K(x, y)g(y)dy$$

for some continuous function  $g: [0, 1] \rightarrow [-1, 1]$ . Prove that the family  $\mathcal{F}$  is equicontinuous.

2. Problem 16 from page 168.

### Part 2

3. Problem 20 from page 169.
4. Problem 24 from page 170.

### Part 3

5. Problem 1 from page 196.
6. Let  $f: [0, 1] \times [0, 1] \rightarrow \mathbb{R}$  be continuous, and suppose  $g_y(x) = f(x, y)$  is continuously differentiable with respect to  $x$ , and  $g'_y(x)$  is continuous with respect to  $y$ . Prove that

$$\frac{d}{dx} \int_0^1 f(x, y)dy = \int_0^1 g'_y(x)dy.$$