PRACTICE NUMBER 1 FOR TEST 2
18.100B SPRING 2007

This test is closed book, no books, papers or notes are permitted. You may use theorems, lemmas and propositions from the class and book. Note that where $\mathbb{R}^k$ is mentioned below the standard metric is assumed.

Problem 1 Put $B = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 1\}$ and suppose $g : B \longrightarrow \mathbb{R}$ is a continuous function. Show that the function

$$f(x, y) = x^2 + y^2 + (x^2 + y^2)(1 - (x^2 + y^2))g(x, y)$$

is continuous on B and prove that there is some point $(\bar{x}, \bar{y}) \in B$ such that $f(\bar{x}, \bar{y}) = \frac{1}{2}$.

Problem 2 If $f : [0, 1] \longrightarrow [0, \infty)$ is increasing and $f(\frac{1}{2}) > 1$, show that $\int_0^1 f(x)dx > \frac{1}{2}$.

Problem 3 Show that if f is a continuous function on $[a, b]$ then there exists a function $g : [a, b] \longrightarrow \mathbb{R}$ such that $g' = f$.

Problem 4 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ are two functions which are continuous at 0, show that the function

$$h(x) = \max\{f(x), g(x)\},\ x \in \mathbb{R}$$

is also continuous at 0.

Problem 5 Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a differentiable functions which satisfies

$$f(x) = 0,\ \text{for } |x| > 10,\ f(-1) = 1,\ f(1) = -1.$$

Show that there are at least two values of $x \in \mathbb{R}$ such that $f'(x) = 0$.