HOMEWORK FOR 18.100B AND 18.100C, SPRING 2007
ASSIGNMENT 2: DUE THURSDAY 15 FEB, AT 11:00 IN 2-108.

(1) (10 pts total) Rudin, Chapter 2, problems 2,3,4. Solutions to these problems can be found at http://ocw.mit.edu/OcwWeb/Mathematics/18-100BAnalysis-IFall2002/Assignments/ so the main point here is to be clear, concise and correct!

(2) (10 pts) Show the supremum of the set $A = \{\frac{x}{\sqrt{1+x^2}}; x \in \mathbb{R}\}$ exists and find it (you need to prove that your answer is correct).

(3) (10 pts) Suppose A and B are bounded, non-empty, subsets of $\mathbb{R}$ and that $A \cap B \neq \emptyset$. Show that

$$\sup(A \cap B) \leq \min(\sup A, \sup B).$$

(4) (10 pts) Show that the set of all rational sequences $a : \mathbb{N} \longrightarrow \mathbb{Q}$ is uncountable. Prove that the subset of terminating sequences, i.e. such that $a_n = 0$ for $n > N$ where N may vary with the sequence, is countable.

Extra problem: This is for your amusement, not to be handed in.

(1) Two fields are *isomorphic* if there is a 1-1 onto map between them which takes sums to sums and products to products. Show that such a map must take 0 to 0 and 1 to 1. Show that the set of real numbers of the form $r+s\sqrt{2}$, where r, $s \in \mathbb{Q}$, is a field. Is it isomorphic to $\mathbb{Q}$?