Problem Set. Modeling airborne disease transmission. 18.085, IAP 2024

In a well-mixed room, small, pathogen-bearing droplets may be suspended by the ambient circulation. Their concentration C determines the infectivity of the air, and so the rate of transmission from infected to susceptible individuals. For the sake of simplicity, consider drops of uniform size suspended in a well-mixed room of volume $V = AH$, height H and area A. The droplet-borne pathogen concentration $C(t)$ evolves with time t according to

$$V\frac{dC}{dt} = p_m IP - (Q + p_f Q_r + v_s A + \lambda_v V)C \tag{1}$$

where I is the number of infected individuals in the room, P is the pathogen production rate per infected individual, $p_m < 1$ is the probability of pathogen filtration by the mask of an infected individual, Q is the rate of exchange between the the contaminated indoor air and the fresh air outside, Q_r is the air recirculation rate, $p_f < 1$ is the probability of filtration by this air recirculation, v_s is the droplet settling speed, and λ_v is the natural rate of pathogen deactivation.

a) If a single infected individual ($I = 1$) enters the room where there are N uninfected people at time $t = 0$, deduce the pathogen concentration $C(t)$. Deduce both the final equilibrium concentration C_{eq} arising in the long-time limit, and the characteristic time of relaxation to this equilibrium value.

b) Transmission arises when an individual has inhaled a sufficient amount of airborne pathogen. The transmission rate to a single individual may be expressed as $\beta(t) = p_m s Q_b C(t)$, where $s < 1$ is the susceptibility of that individual, and Q_b is the breathing rate. Calculate the *indoor reproductive number*, $R_{in}(\tau)$ of the virus, specifically, the mean number of transmissions after a time τ:

$$R_{in}(\tau) = N\int_0^\tau \beta(t)dt \tag{2}$$

c) Comment on the utility of masks in reducing airborne transmission. Specifically, why are they such an effective means of reducing transmission?

Additional reading

1) Bourouiba, L., Dehandschoewercker, E. and Bush, J. W. M., 2014. Violent expiratory events: on coughing and sneezing, *J. Fluid Mech.*, **745**, 537-563.

2) Bazant, M.Z. and Bush, J.W.M., 2021. A guideline to limit indoor airborne transmission of COVID-19, *Proc. Nat. Acad. Sci.*, **118** (17), doi:e2018995118.

3) Bazant, M.Z , Kodio, O., Cohen, A.E., Khan, K., Gu, Z. and Bush, J.W.M., 2021. Monitoring carbon dioxide to quantify the risk of indoor airborne transmission of COVID-19, *Flow*, **1**, E10.