In a well-mixed room, small, pathogen-bearing droplets may be suspended by the ambient circulation. Their concentration $C$ determines the infectivity of the air, and so the rate of transmission from infected to susceptible individuals. For the sake of simplicity, consider drops of uniform size suspended in a well-mixed room of volume $V = AH$, height $H$ and area $A$. The droplet-borne pathogen concentration $C(t)$ evolves with time $t$ according to

$$V \frac{dC}{dt} = p_m IP - (Q + p_f Q_r + v_s A + \lambda_v V)C$$

where $I$ is the number of infected individuals in the room, $P$ is the pathogen production rate per infected individual, $p_m < 1$ is the probability of pathogen filtration by the mask of an infected individual, $Q$ is the rate of exchange between the the contaminated indoor air and the fresh air outside, $Q_r$ is the air recirculation rate, $p_f < 1$ is the probability of filtration by this air recirculation, $v_s$ is the droplet settling speed, and $\lambda_v$ is the natural rate of pathogen deactivation.

**a)** If a single infected individual ($I = 1$) enters the room where there are $N$ uninfected people at time $t = 0$, deduce the pathogen concentration $C(t)$. Deduce both the final equilibrium concentration $C_{eq}$ arising in the long-time limit, and the characteristic time of relaxation to this equilibrium value.

**b)** Transmission arises when an individual has inhaled a sufficient amount of airborne pathogen. The transmission rate to a single individual may be expressed as $\beta(t) = p_m s Q_b C(t)$, where $s < 1$ is the susceptibility of that individual, and $Q_b$ is the breathing rate. Calculate the indoor reproductive number, $R_{in}(\tau)$ of the virus, specifically, the mean number of transmissions after a time $\tau$:

$$R_{in}(\tau) = N \int_0^\tau \beta(t) dt$$

**c)** Comment on the utility of masks in reducing airborne transmission. Specifically, why are they such an effective means of reducing transmission?

**Additional reading**

