Numerical simulations with exponential accuracy

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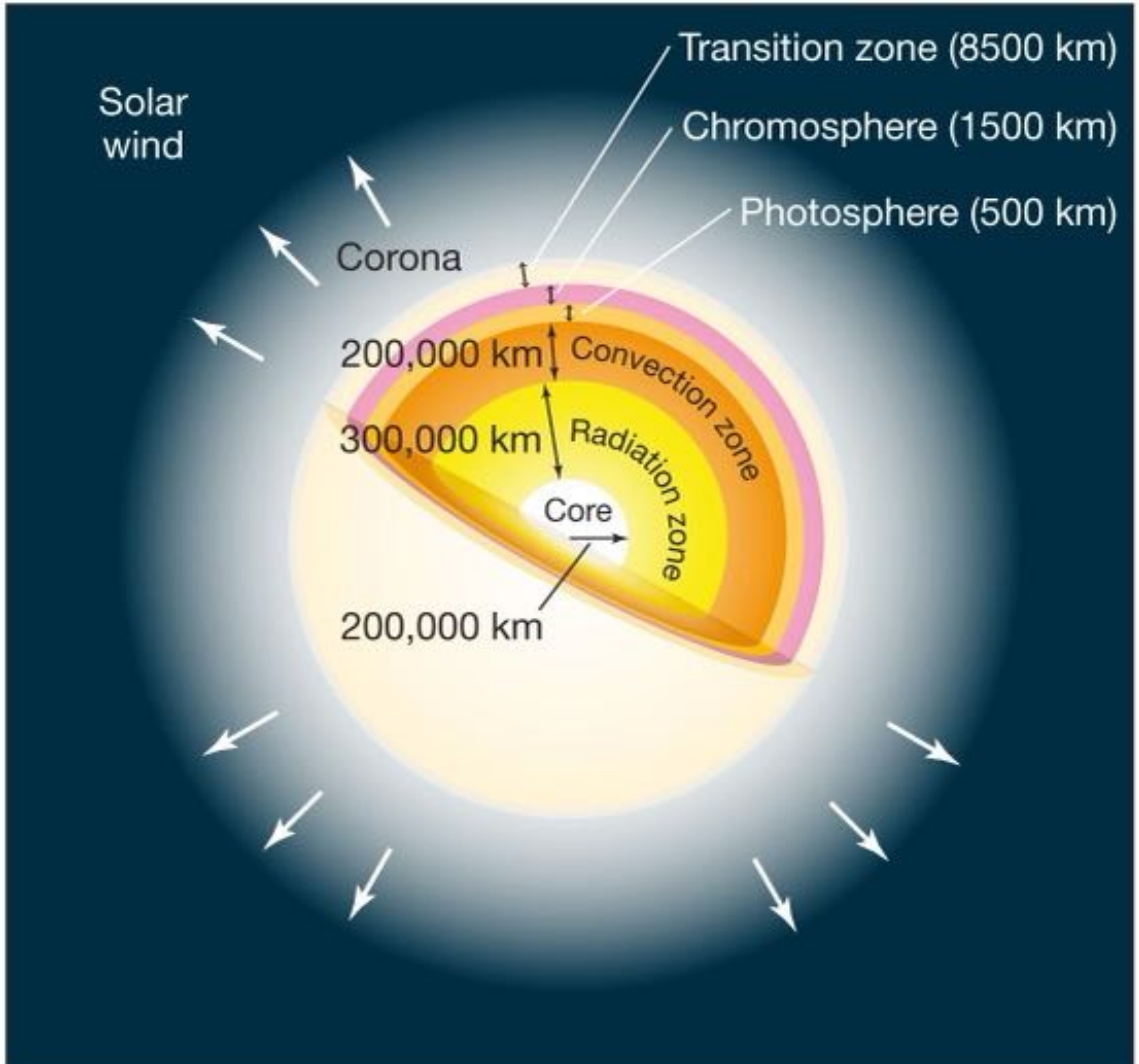
18.095 IAP 2024

1. Motivation
2. Review finite differences for PDEs
3. Fourier spectral methods
4. Polynomial spectral methods

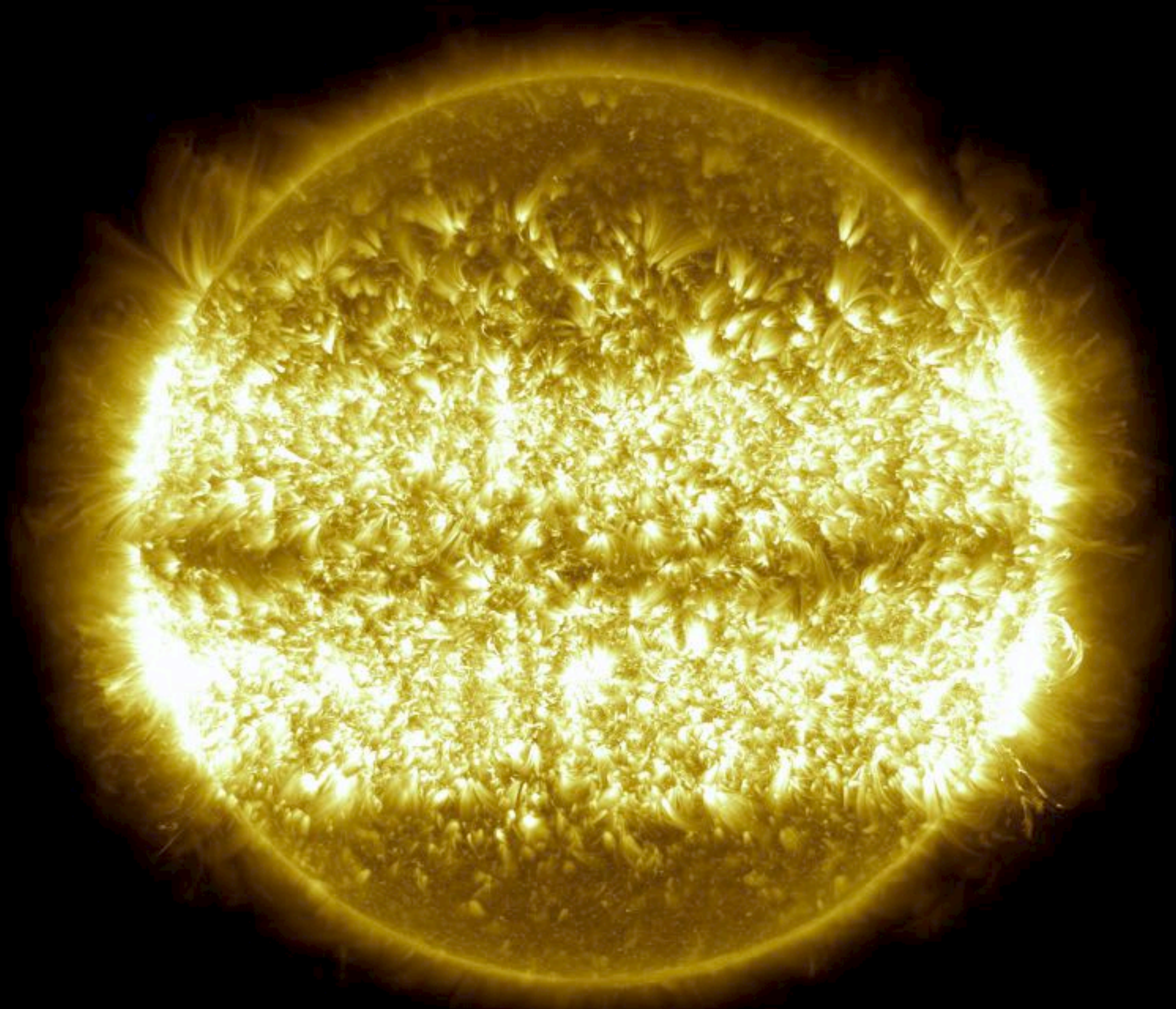
Some Motivation: Fluid Dynamics

Inside the sun



Solar Dynamics Observatory

Solar Dynamics Observatory



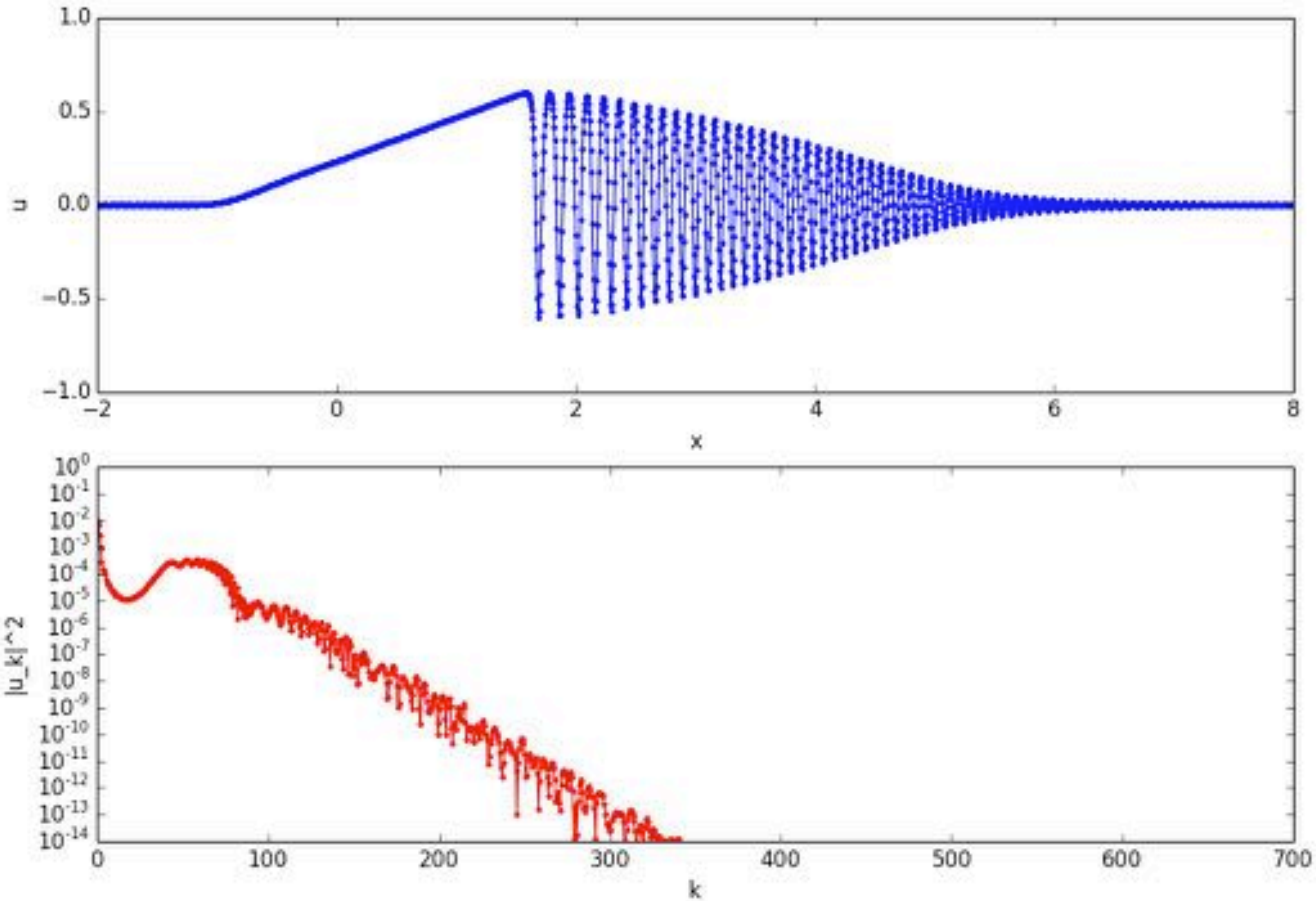
ECCOv2 ocean simulation (MIT GCM)

Scientific American & David H. Laidlaw

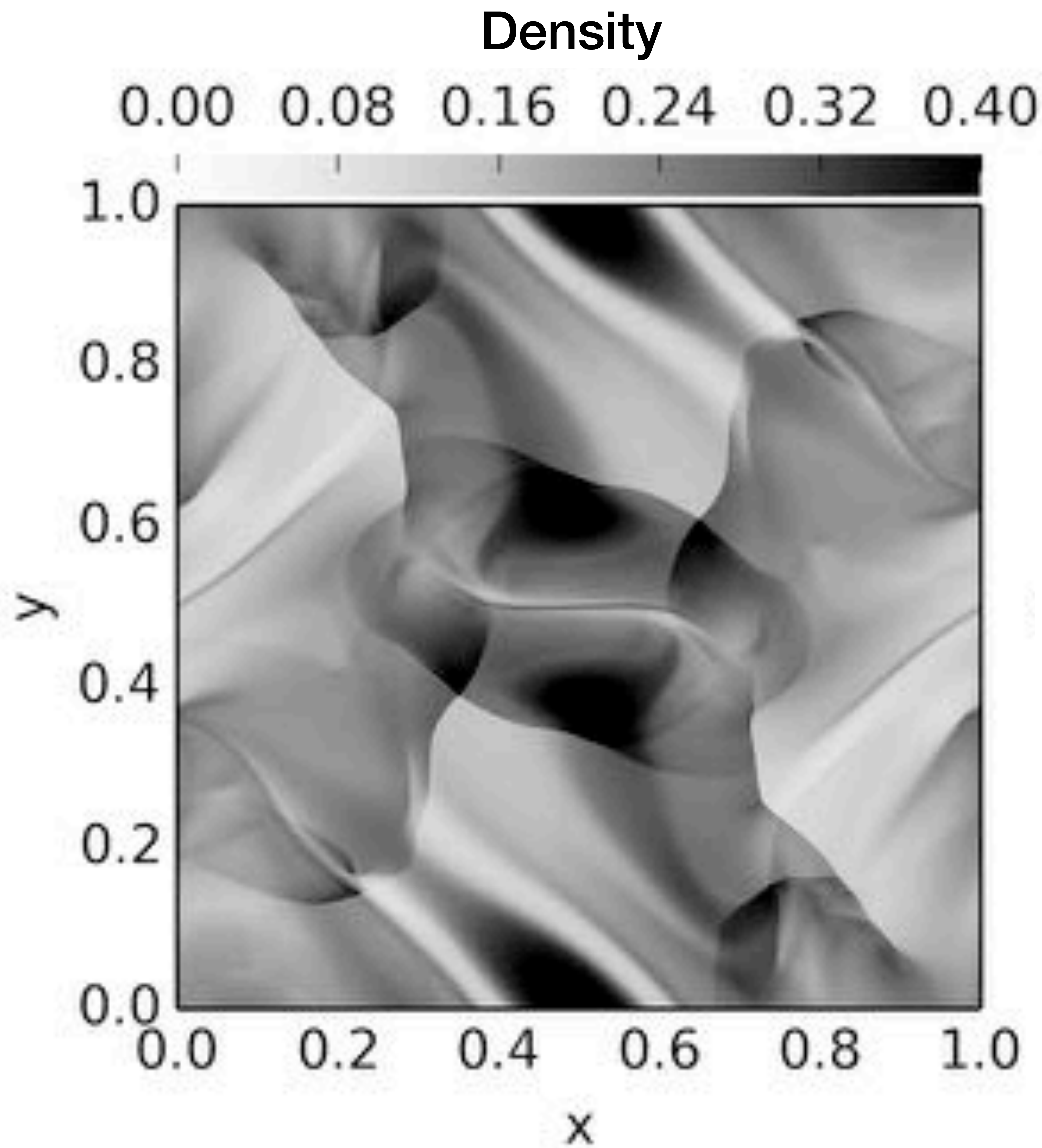
Fourier Spectral Methods

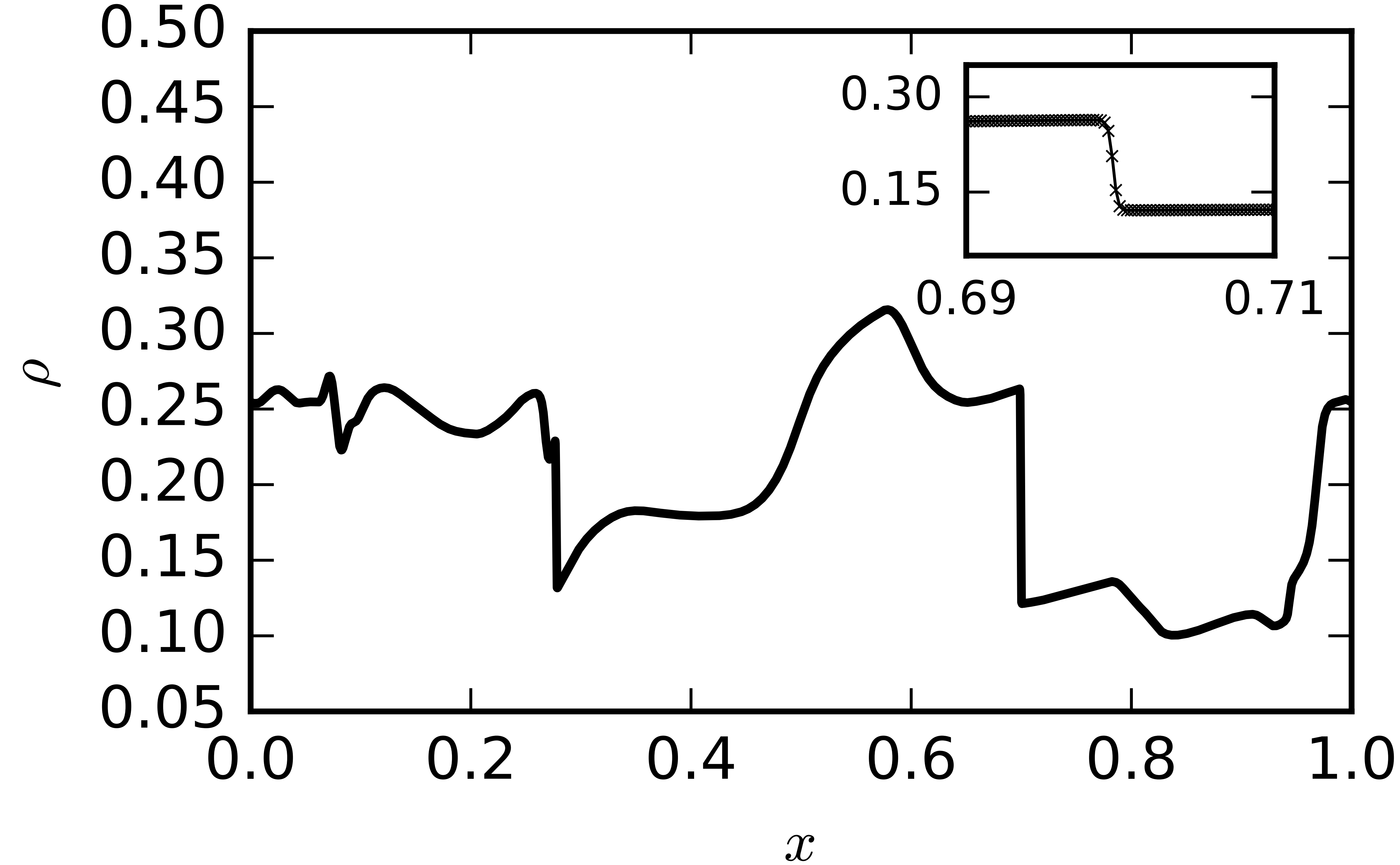
KdV dispersive shock

$$\partial_t u + \partial_x^3 u = -u\partial_x u$$



Viscously regularized MHD shocks

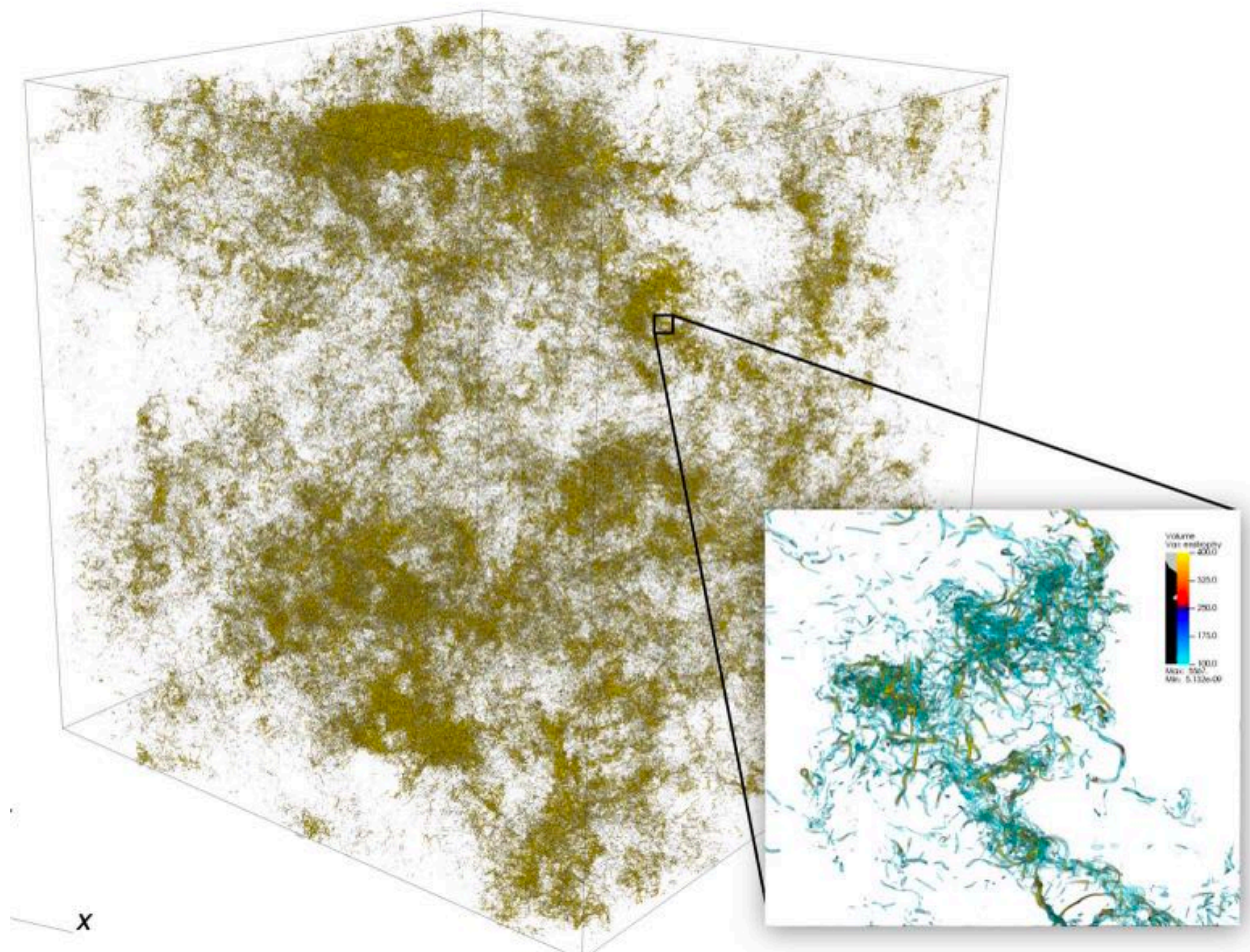




World's largest turbulence simulations (not Dedalus)

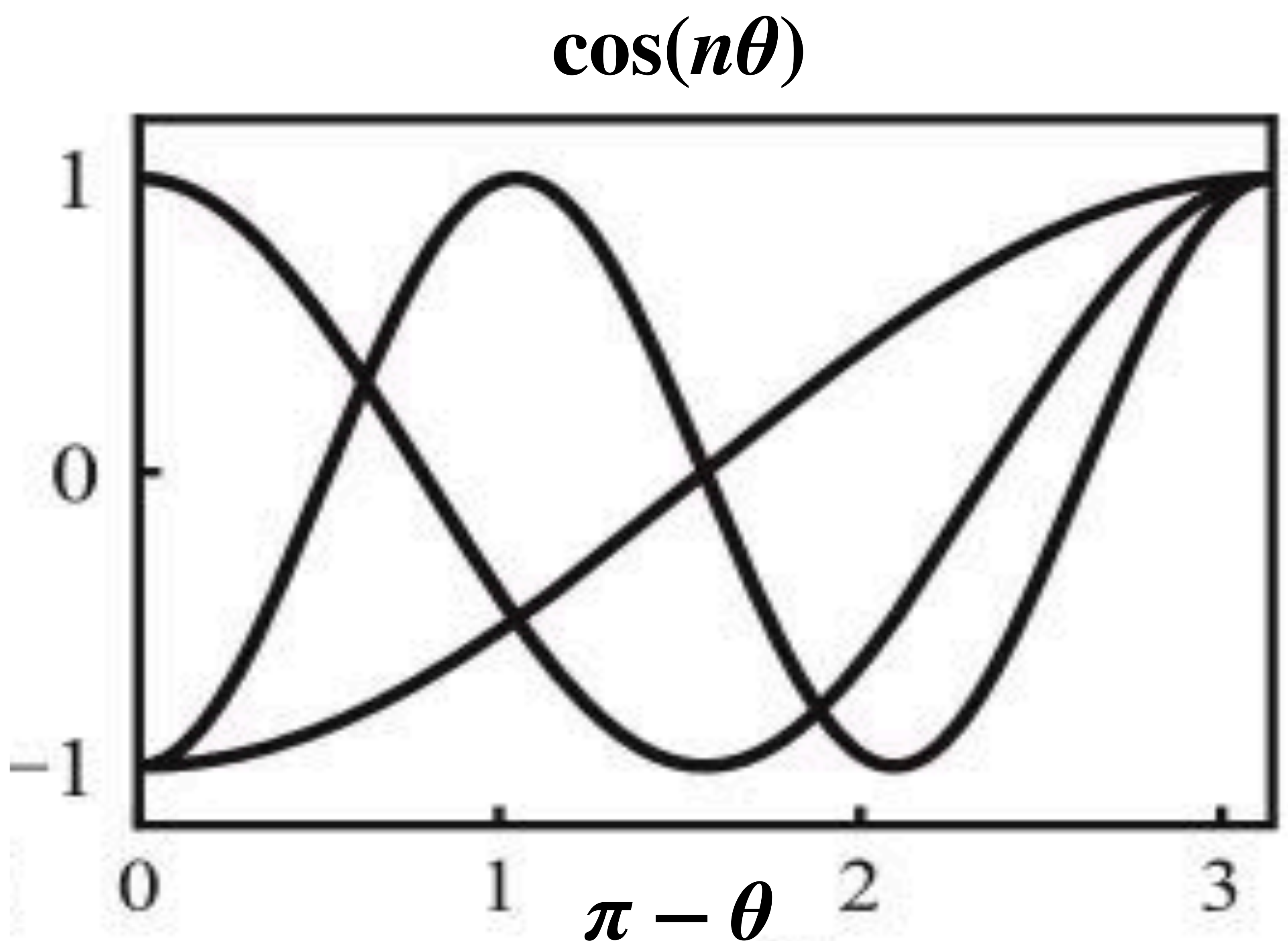
Yeung & Ravikumar, Phys. Rev. Fluids (2021)

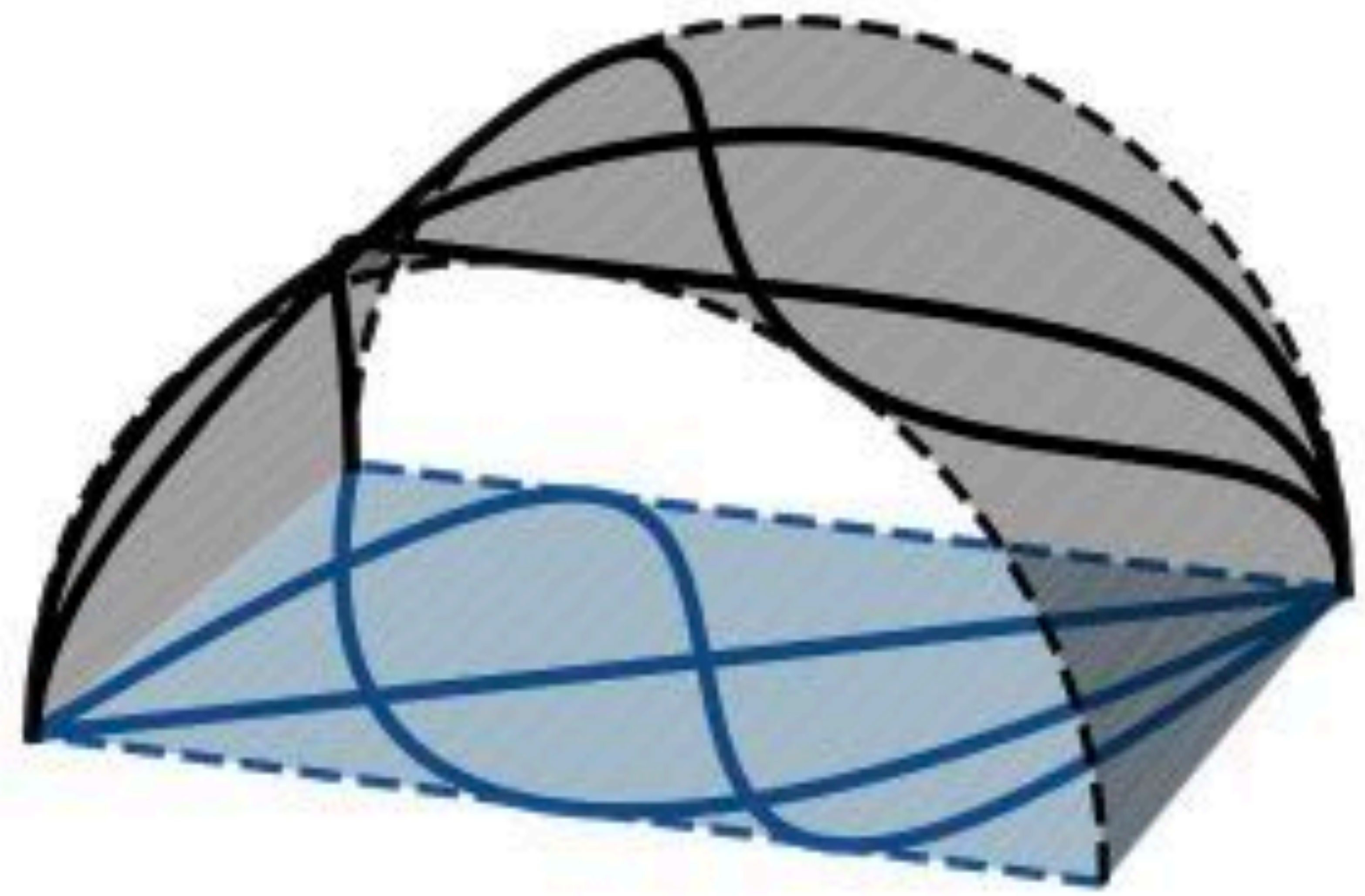
- Fourier pseudospectral method
- $18{,}432^3$ grid points
- 18,432 GPUs

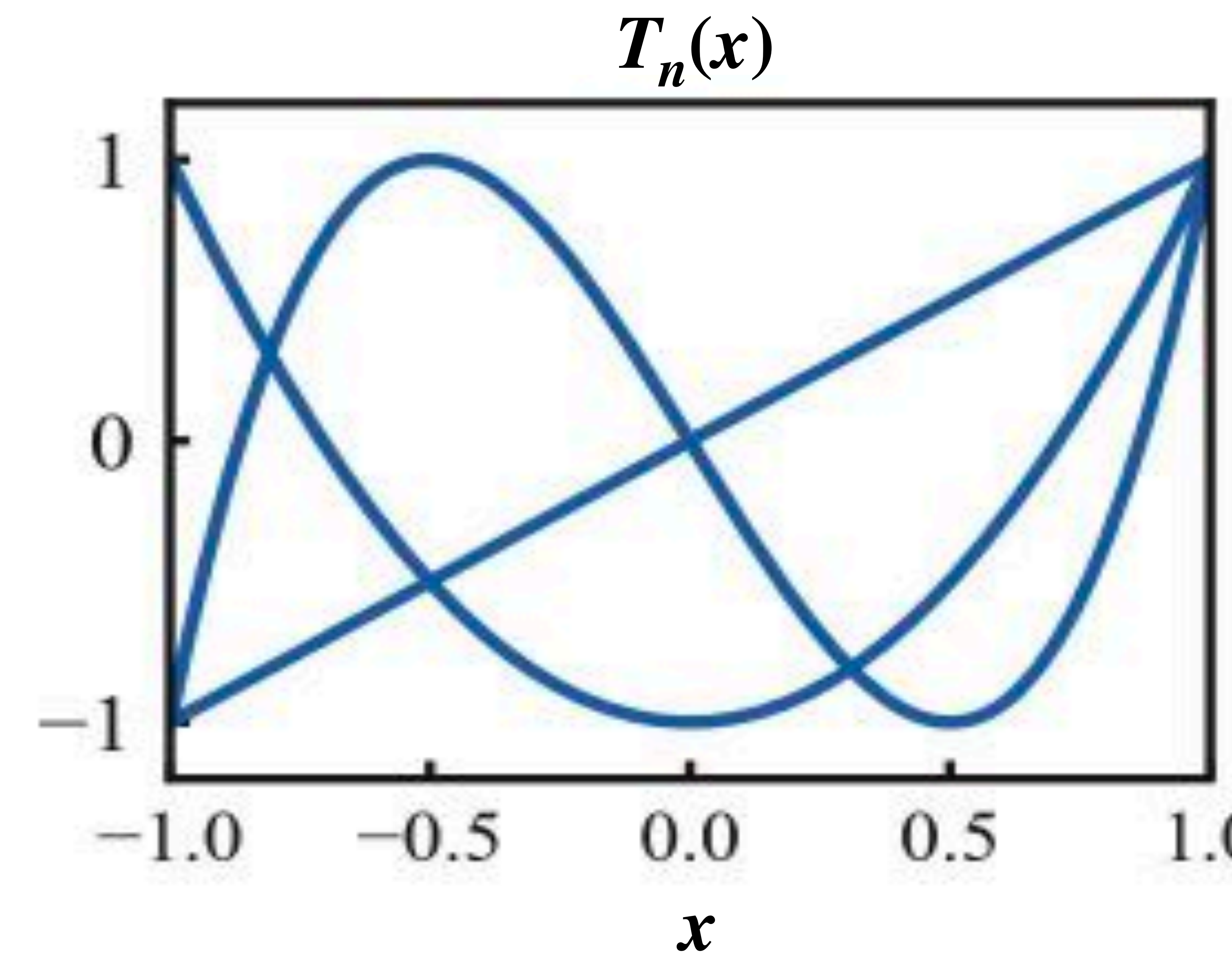


Chebyshev Spectral Methods

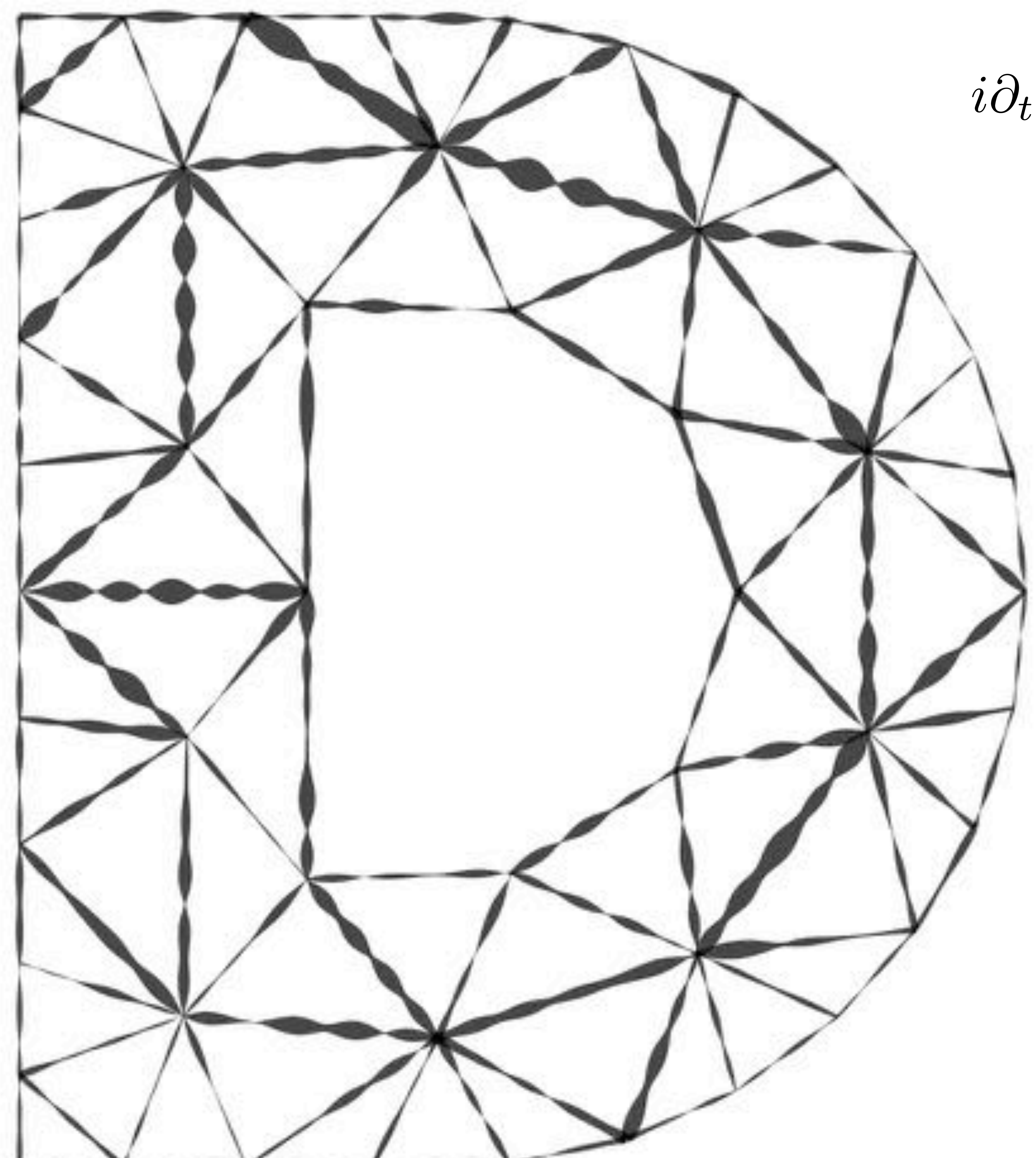
Chebyshev polynomials: cosines in disguise







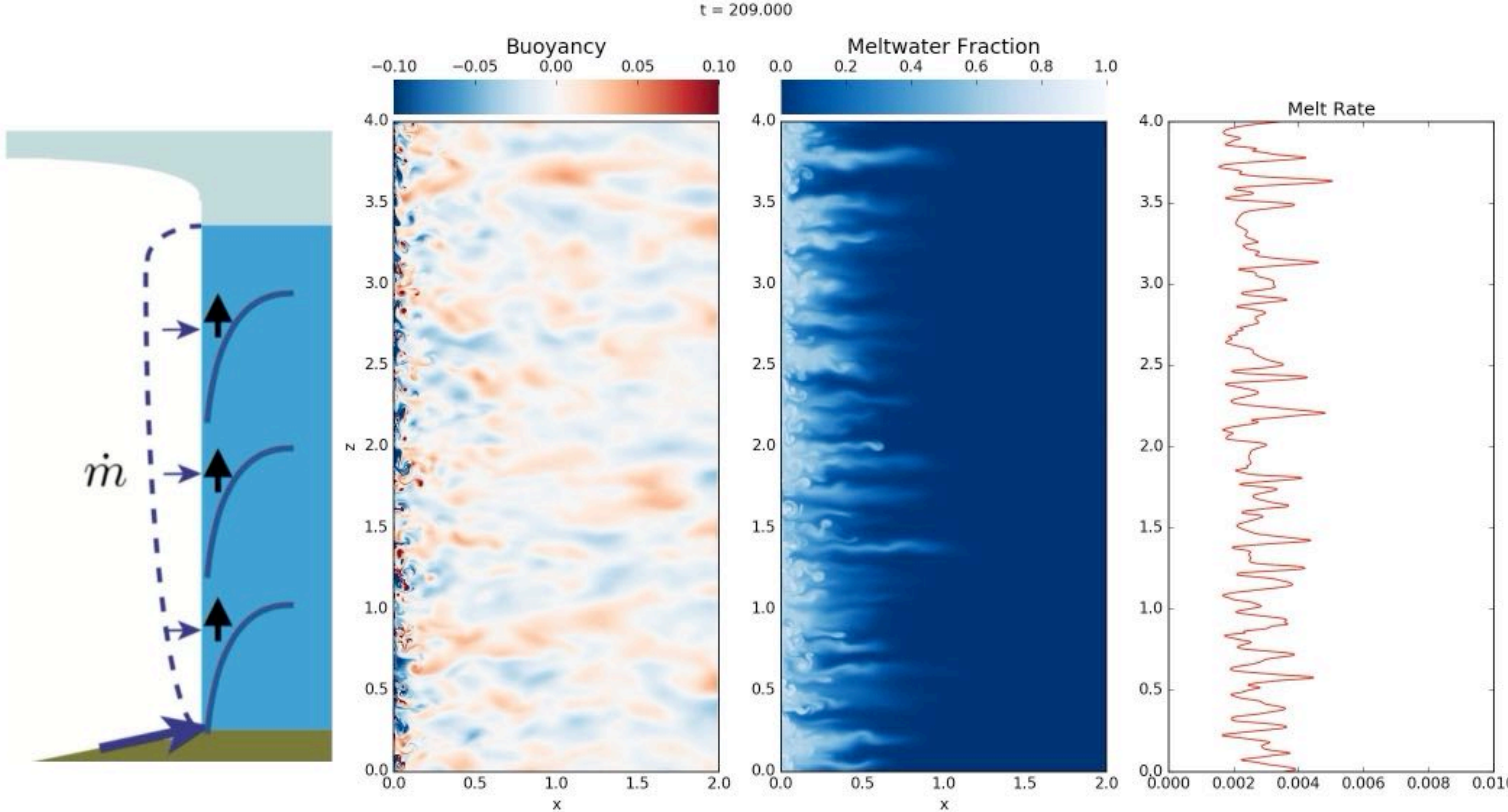
Quantum graphs



$$i\partial_t\psi + \frac{1}{2}\partial_x^2\psi = -|\psi|^2\psi$$

Burns et al., PRR 2020

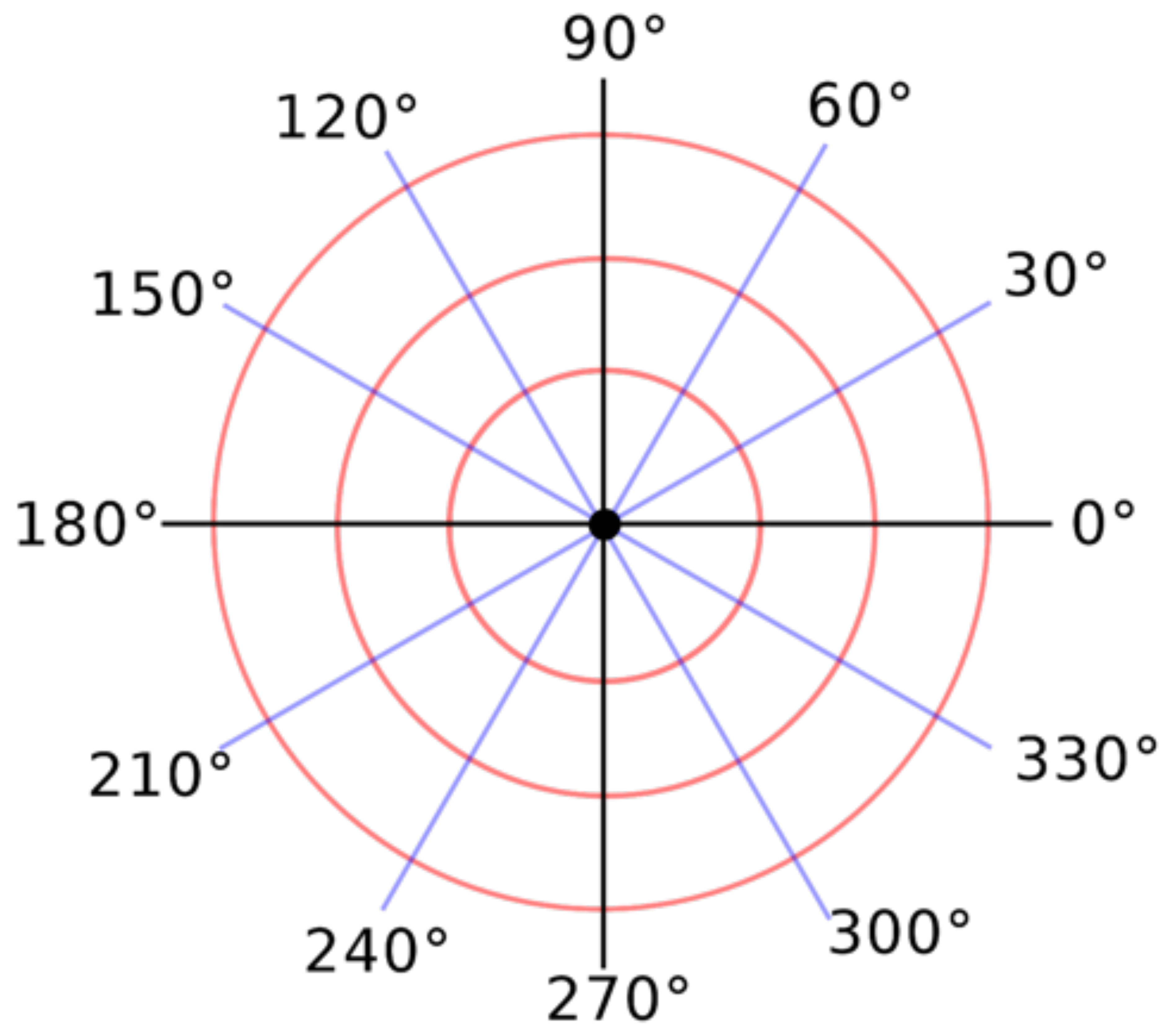
Turbulent enhancement of glacier melting

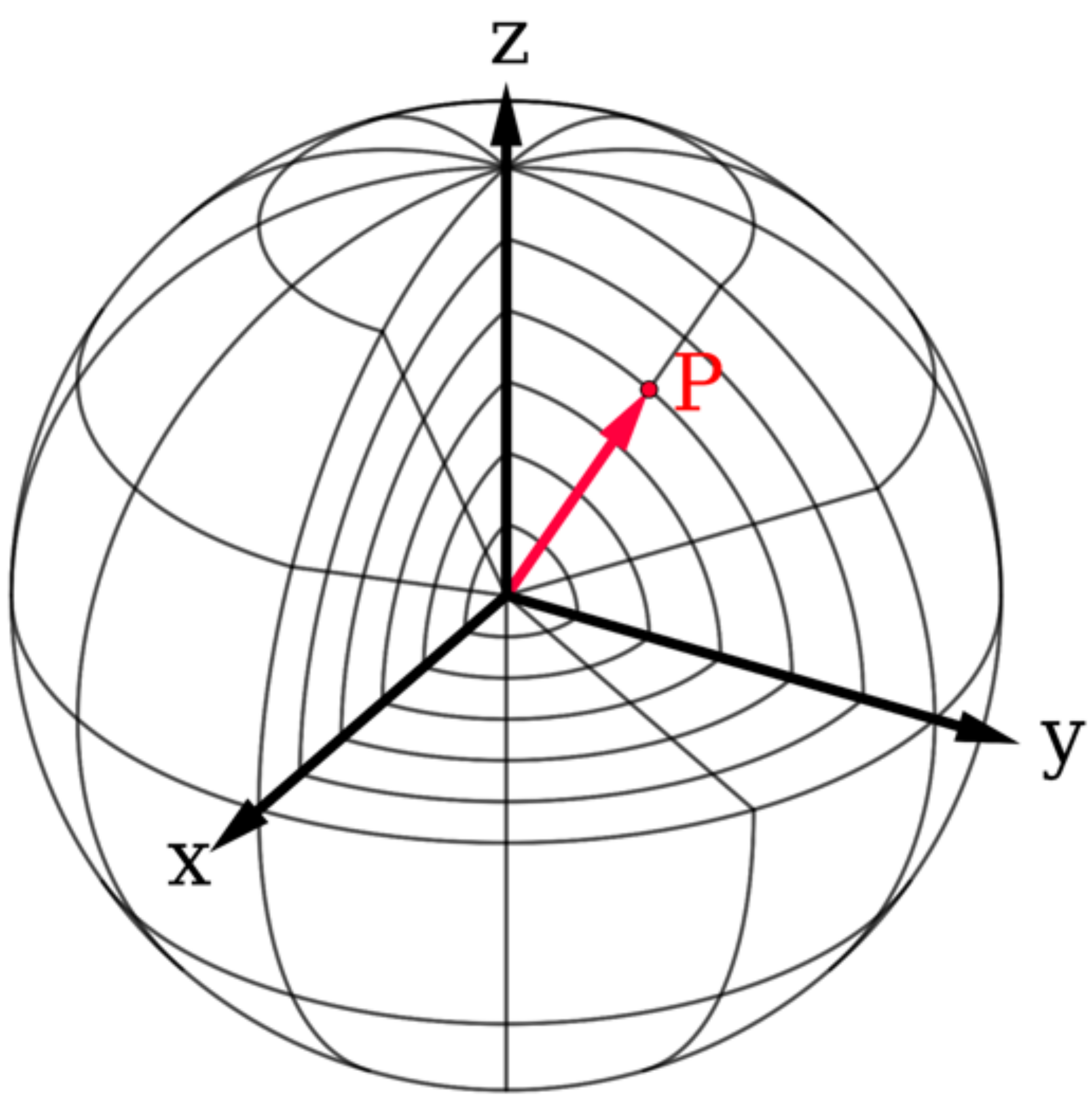


Burns et al., in prep

Curvilinear Spectral Methods

Polar & spherical coordinate singularities





Active matter turbulence

Mickelin et al., PRL (2018)

Incompressible hydrodynamics

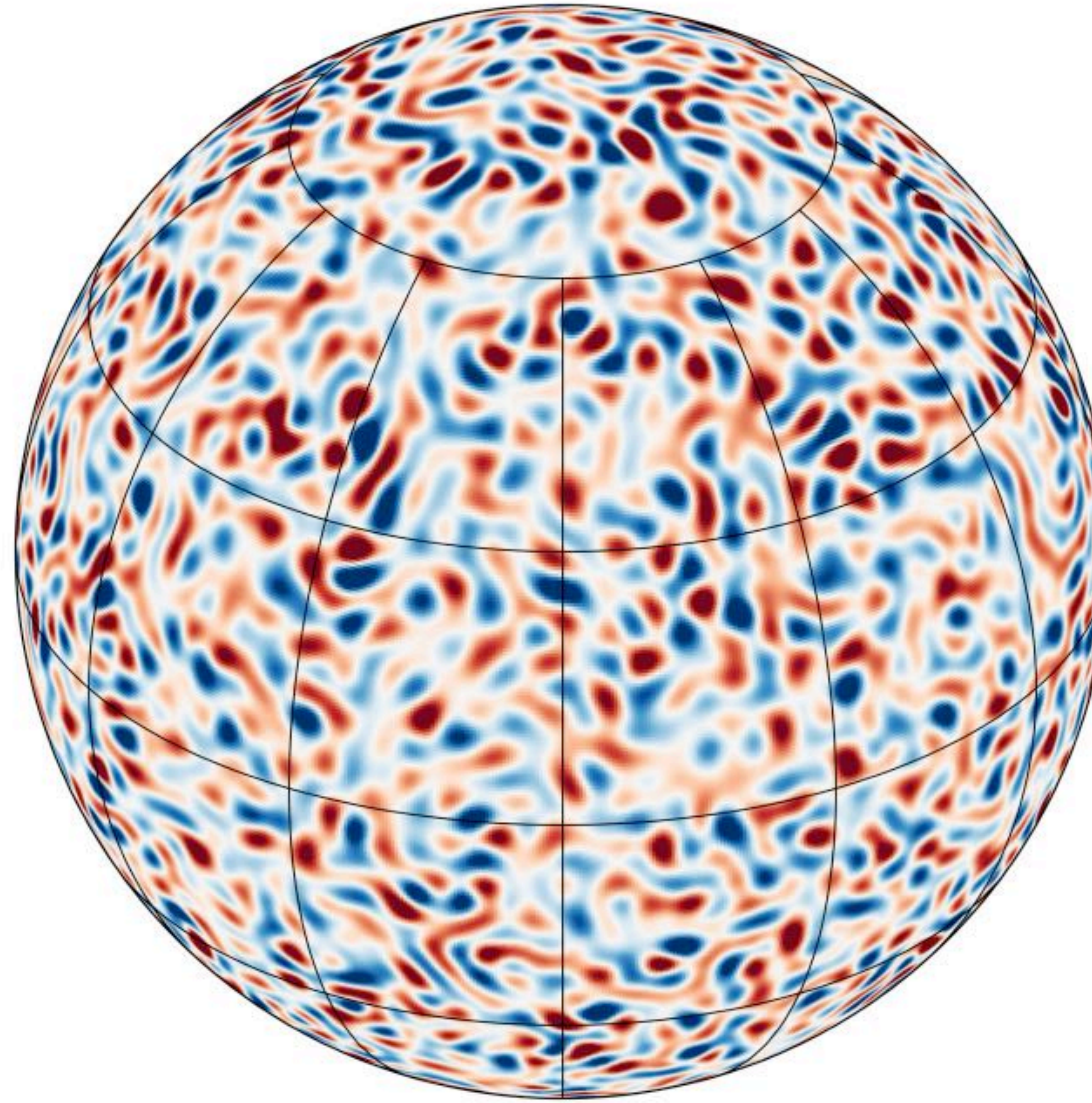
+ linearly unstable bandpass:

$$T^{ab} = f(\nabla^2)(\nabla^a u^b + \nabla^b u^a)$$

$$f(\nabla^2) = \Gamma_0 - \Gamma_2 \nabla^2 + \Gamma_4 \nabla^2 \nabla^2$$

Supekar et al., JFM (2020)

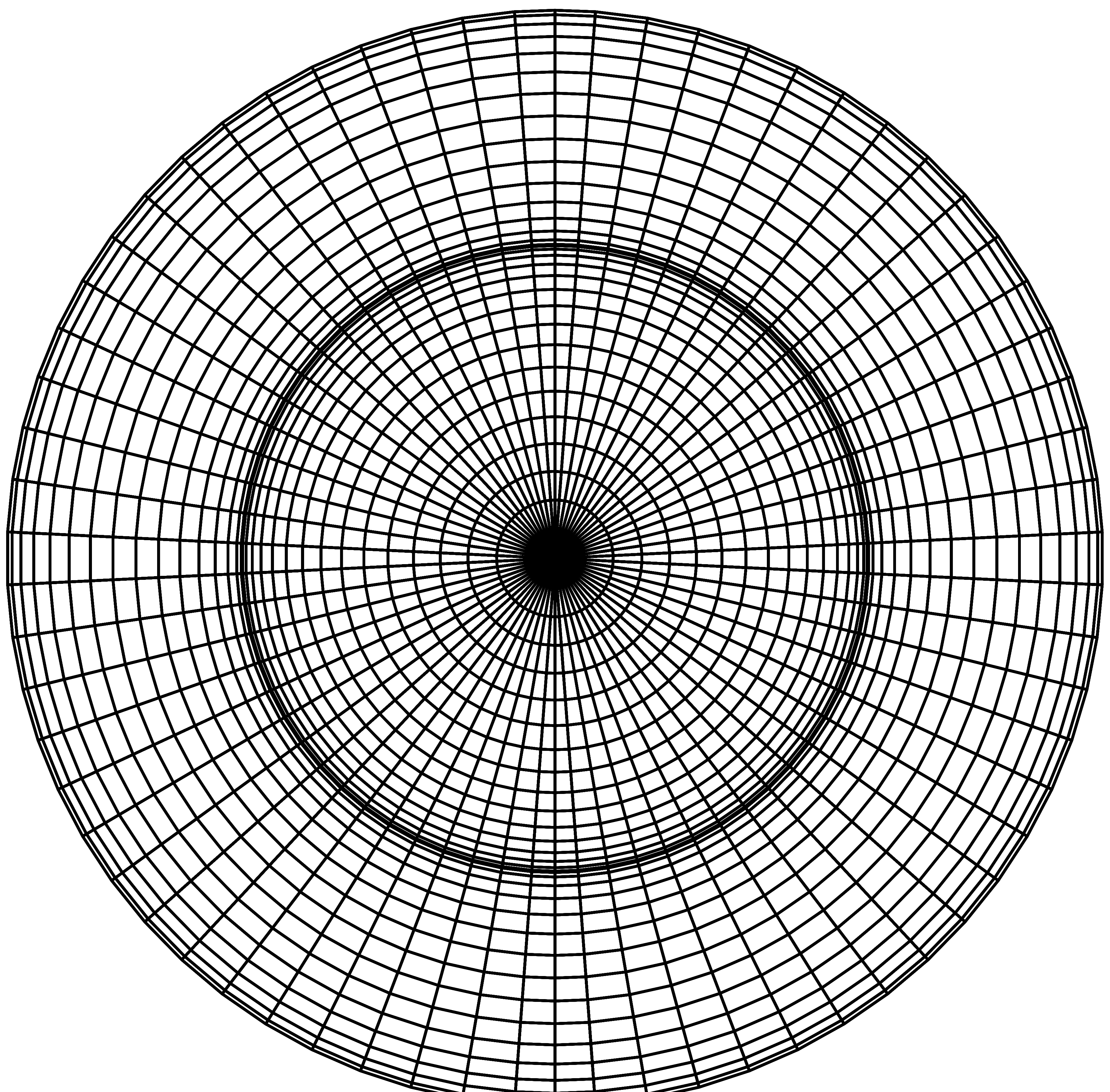
- Added rotation
- Rossby wave turbulence

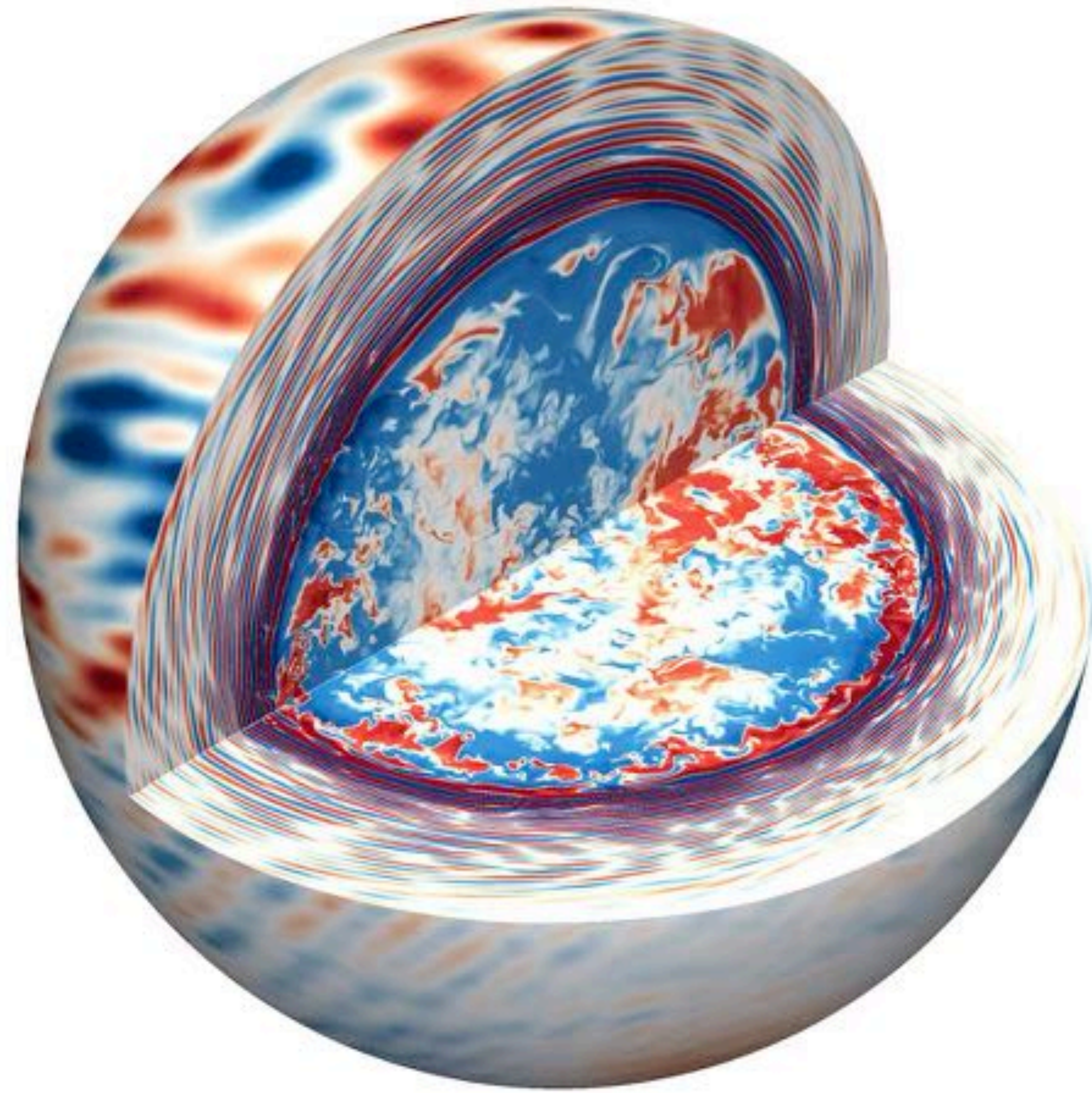


Mickelin et al. PRL (2018)

High-*p* spherical spectral elements

- Stacked ball and spherical shell bases
- Resolves internal/material boundaries





Anders et al., Nature Astronomy (2023)