18.095: Mathematics Lecture Series, IAP 2024

Random Walks, Discrete Harmonic Functions, and Electrical Circuits

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Reference

• Today’s lecture draws heavily from *Random walks and electric networks* by Doyle and Snell
  – Available online at [http://tinyurl.com/2w7tjjp](http://tinyurl.com/2w7tjjp)
  – Really excellent monograph containing what we cover and much more. I highly recommend it.
Classic problem in probability: Drunkard’s Walk

• Goes left or right with prob. 1/2 each step
• Stops when gets to either bar or house
• If starts at node $i$, what is prob gets home before bar?
• What if sort of knows which way house is, and goes right w/ prob. 0.6, left w/ prob. 0.4?
• Could ask same kind of thing in different ways, e.g.,
  – Go to a casino, starting with $100, repeatedly gamble $1 per hand on blackjack.
  – What is the probability that you triple your money before you go broke?
  – How does this probability change if you count cards?

Will Modify the Question

Why?
• Make it a bit harder
• Don’t want to leave the impression theory just about drinking and gambling
• After finishing a pset, sleep deprived student wanders the MIT tunnels, starting under building 2
• Wants to go home to building 62, so leaves tunnels at 54
• If reaches Stata Center, gets disoriented and passes out
• What is prob. makes it home before passing out?
• If don’t say otherwise, go to each neighbor with same probability, until reach absorbing state
• If want different probs, add **edge weights**, choose edge with prob. \( \propto \) weight
Quick Review of Electrical Networks

- Will show how to relate these questions to electrical networks
- Will give a very quick review of everything you’ll need to know about electrical networks
Quick Review of Electrical Networks

- Graphs will become circuits

Edge \( e \) with weight \( w_e \) ↔ Resistor with res. \( r_e = 1/w_e \)

Vert. \( a \) has a potential (voltage) \( v_a \)

Edge \( e \) has a current \( i_e \)

For \( e = (a, b) \), current and voltage obey

\[
  i_e = \frac{v_a - v_b}{r_e}
\]

Have **internal nodes** and **boundary nodes**

At internal nodes, total current in = total current out