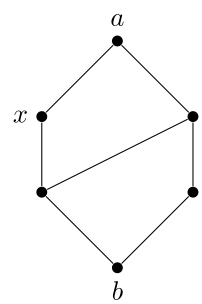
Problem 1: Consider the following modified version of the Drunkard's Walk: As before, there are $n+1$ vertices, labeled $0, \ldots, n$, and the walker stops if he ever reaches either vertex 0 or vertex n. However, now the walker will not be equally likely to go in either direction. If he is at node $i = 1, \ldots, n-1$, he will go to node $i+1$ with probability p and to node $i-1$ with probability $1-p$.

(a) Find weights on the edges so that this corresponds to the type of walk we studied in class (where, the walker follows each outgoing edge of the present vertex with probability proportional to its weight). Remember that the edge (x, y) has a single weight $w_{x,y}$ that is used to determine both the probability of going to y if the walker is at x, and the probability of going to x if the walker is at y.

(b) Describe the corresponding electrical network. Use this to compute the probability that, if the walker starts at vertex i, he gets to vertex n before vertex 0 (and say what fact about the electrical network this corresponds to).

Problem 2: Consider the graph shown below, where all edges have weight 1. Compute the probability that a random walk started at node x reaches node a before it reaches node b.



Problem 3: Let a and b be two nodes in an electrical network. Fix the voltages $v(a) = 1$ and $v(b) = 0$, and let $\alpha = \sum_{x \sim a} i_{ax}$ be the total number of units of current flowing out of a. We call α the *effective conductance* between a and b, and we call $1/\alpha$ the *effective resistance* between a and b. (See the last page of the lecture notes for how effective resistance corresponds to the probabilistic interpretation of current.)

Define the *escape probability* p_{esc} to be the probability that a particle starting at a reaches b before returning to a. Show that the effective conductance between a and b equals $p_{\text{esc}} \cdot \deg(a)$.

[The answer to this question is given in Doyle and Snell. Please try to solve the problem without looking it up.]