Continued Fractions Problems Peter Shor IAP2024

1. Find the continued fraction for
$$\alpha = \frac{\sqrt{10}-2}{3}.$$

2. Find a fraction $\frac{r}{s}$ that is a good approximation for $\alpha = \frac{\sqrt{10}-2}{3}$ with s at most 50. The difference between $\frac{r}{s}$ and α should be at most 0.0005.

3. Show that any continued fraction of the form
$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{\ddots \cfrac{1}{a_k + x}}}}},$$
reduces to an expression of the form
$$\frac{rx+s}{tx+u}$$

4. In class (and the lecture notes) we showed that any continued fraction for a quantity of the form
$$\frac{\sqrt{d}-p}{q}$$
with $p, q, d > 0$, was eventually periodic. Show this is also true for the continued fractions for quantities of the form
$$\frac{p-\sqrt{d}}{q} \quad \text{and} \quad \frac{p+\sqrt{d}}{q}.$$
You may assume that we only consider continued fractions for positive numbers, and that p, d, q are positive.

Hint: Show that when you are computing the continued fraction, the quantity $\left|\frac{p}{\sqrt{d}}\right|$ decreases with each step until you reach a fraction of the form $\frac{\sqrt{d}-p}{q}$.