HOMEWORK 18.095, JANUARY 2024
LECTURE OF 1/31.

(1) Find continued fraction for $\alpha = 1+\sqrt{2}$ and show that the Legendre number $L(\alpha) = \sqrt{8}$.
(2) Let F_n be the n-th Fibonacci number, thus $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$. Show that $F_{2n+1} = 3F_{2n-1} - F_{2n-3}$. Deduce that an odd Fibonacci number is a Markov number.
(3) Let α be a quadratic irrationality with a purely periodic continued fraction, let $\Pi = (a_0, \dots, a_n)$ be the period. Define matrices

$$A_i = \begin{pmatrix} a_i & 1 \\ 1 & 0 \end{pmatrix}$$

and $A = A_0 \dots A_n$. Show that α is the *fixed point* of the fractional linear transformation $x \mapsto \frac{ax+b}{cx+d}$, where

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$