PROBLEM SET: TRIANGLES AND EQUATIONS, IAP 2021

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(1) Let $R(s,t)$ denote the smallest integer n such that every red/blue edge-coloring of K_n contains either a red K_s or a blue K_t (here K_n is the complete graph on n vertices).

Show that, for all integers $s, t \geq 2$,
$$R(s,t) \leq R(s-1,t) + R(s,t-1).$$

Finally, deduce that
$$R(s,t) \leq \binom{r+s-2}{r-1}$$

(2) Prove that it is possible to color the integers using two colors so that there is no infinitely long monochromatic arithmetic progression.