

IAP Mathematics Lecture, 1/25/2021, MIT

Mathematics of COVID-19 Transmission


Martin Z. Bazant

*E. G. Roos (1944) Professor of Chemical Engineering
Professor of Mathematics
Massachusetts Institute of Technology*

How does COVID-19 spread?

Modes of Transmission of Respiratory Pathogens

Key _____

-  **Respirable Aerosol**
≤ 5μm
-  **Thoracic Aerosol**
≤ 10μm
-  **Nasopharyngeal Aerosol**
≤ 100μm
-  **Fomite**

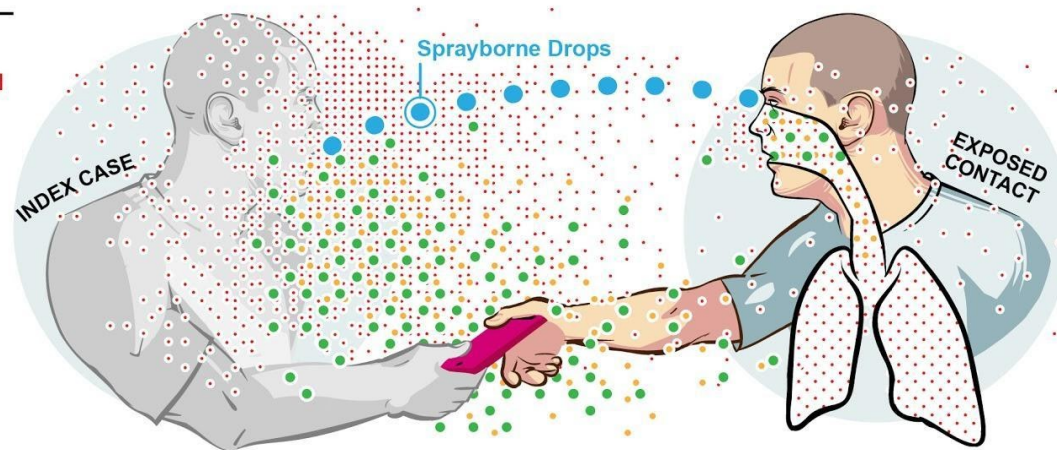
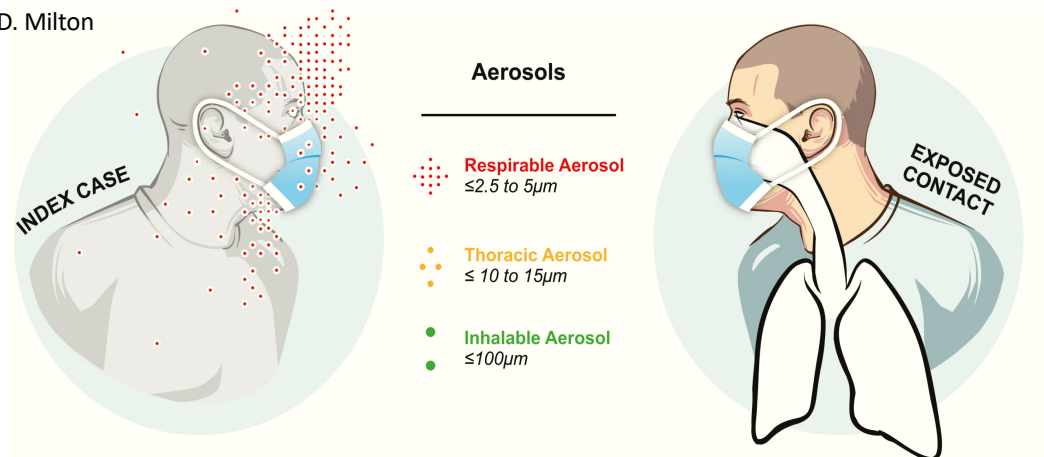


Image credit: D. Milton

1. Fomite / Contact
2. Large / Ballistic Drops
3. Airborne / Aerosol

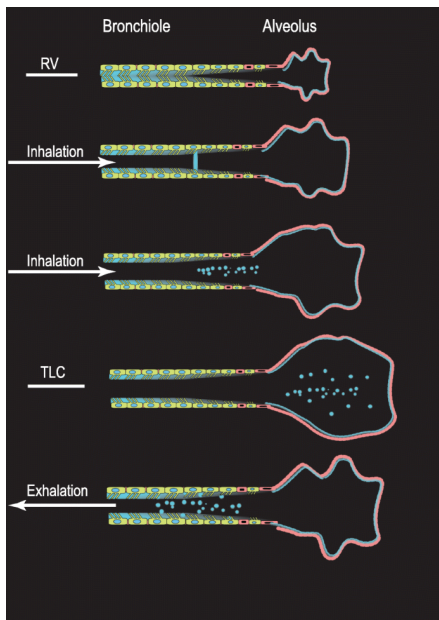
<https://tinyurl.com/FAQ-aerosols>

FAQs on Protecting Yourself from COVID-19 Aerosol Transmission
J. Jimenez, D. Milton, S. Miller, L. Marr, L. Morawska,...



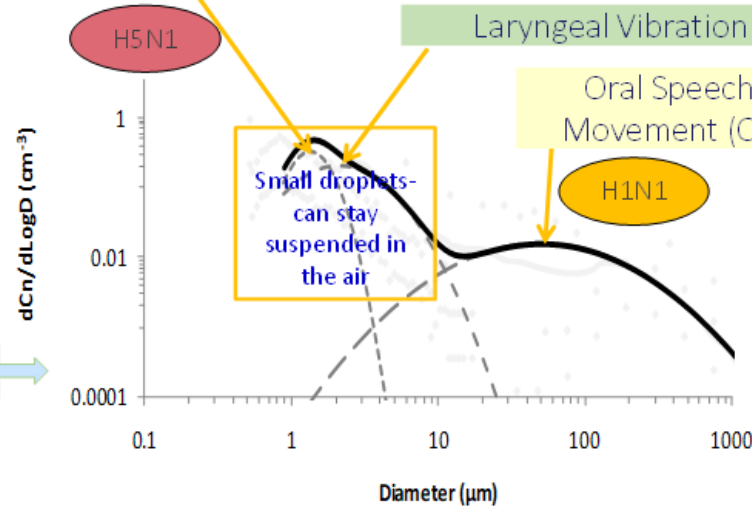
D. Milton, *J Pediatric Infect Dis Soc* (2020)

Mechanisms for Expiratory Droplet Formation



B. Bake et al, Respiratory Research (2019)

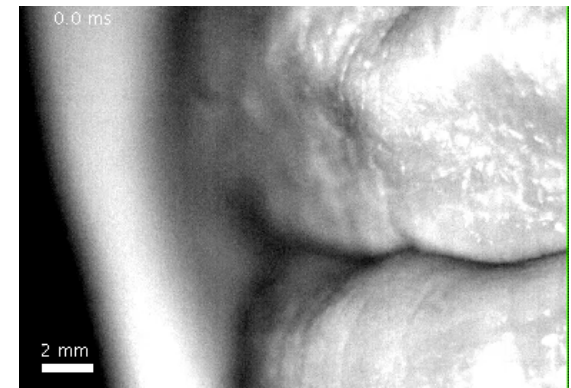
Bronchial Fluid Film Burst Mode (BFFB)



Morawska et al., 2009. Size distribution and sites of origin of droplets expelled during expiratory activities. *Journal of Aerosol Science*, 40: 256-269, 2009.



Glottis (larynx) during speech
James P. Thomas, MD
voicedoctor.net



Abkarian & Stone, Phys. Rev. Fluids (2020)

Fate of Expiratory Drops

- *Pure liquid* drops settle or evaporate (Wells Curve):

Stokes settling time

$$\tau_s = \frac{9\mu L}{2\rho g R^2}$$

Evaporation time

$$\tau_e = \frac{R_0^2}{\bar{D}(1-RH)}$$

- “Droplet nuclei” are *stabilized by solutes* (mucins, salts...) and remain suspended

$$\frac{R_{eq}}{R_0} = \left(\frac{\phi_{solute}^0}{1-RH} \right)^{1/3}$$

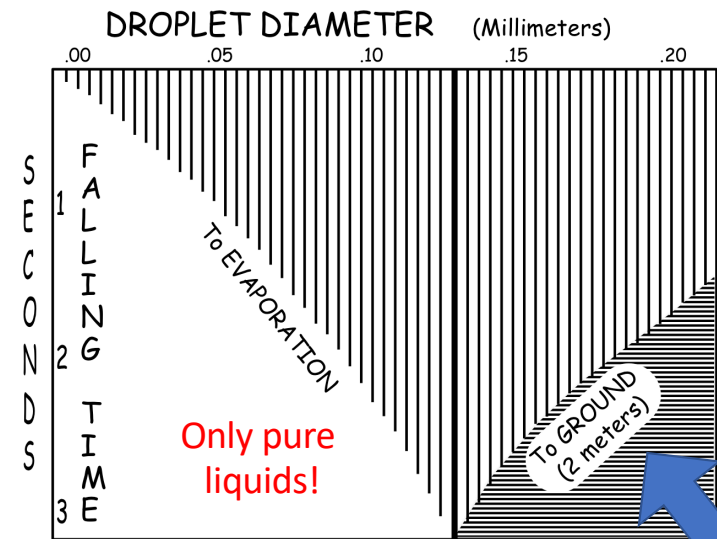
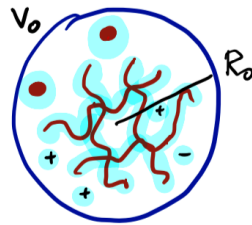


CHART 1. Falling and evaporation times for droplets of varying diameter. Redrawn from Wells, W. F. 1934. wikipedia

(Origin of the 6 Foot Rule!)

Indoor Airborne Transmission: *Overwhelming Evidence*

- All 1200+ super-spreading events indoors, presumed airborne

[SARS-CoV-2 Superspreading Event Database, <https://covid19settings.blogspot.com/p/project.html>]

- Jan. 19, 2020: **Tiangtong Temple, Ningbo, China. 2 buses** 23/68 infected by 1 person on one bus (100min ride), none on the other bus; **no correlation with seating**
- Feb. 3-15, 2020: **Diamond Princess cruise ship. 12-day quarantine** in Yokohama. ~20 cases → **354/3711 infected** on board; **no correlation with shared rooms**
- March 10, 2020: **Skagit Valley Chorale, WA. 53/61 infected** by one in 2.5-hour choir practice (2 died, median age 69). **no correlation with "close contacts"**



- Indoor transmission from Wuhan across China

[Qian et. al, medRxiv (2020).]

- 7324 initial cases in 320 cities outside Hubei Province
- **All 318 outbreaks occurred indoors** (2 or more transmissions)
- **80% in family apartments, 34% involved public transportation**
- Only 1 cluster outdoors (2 transmissions)



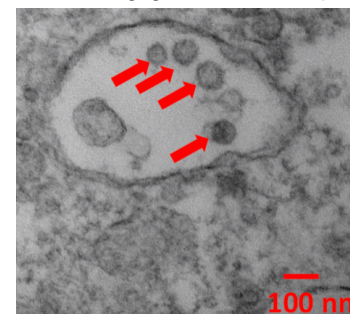
Sanche, Emerging Infectious Diseases (2020)



- Patient-generated infectious aerosols confirmed

[Santarpia et. al, medRxiv (2020). Lednicky et al, Int. J. Infectious Diseases (2020)]

- Collected in hospital rooms **16 feet from patients**
- **SARS-CoV-2 virions present and most infectious in <4um droplets**
- Similar to SARS coronavirus, measles, tuberculosis,...



Santarpia et al, medRxiv (2020)



Aerosol Science vs. Public Health Guidance

The coronavirus pandemic and aerosols: Does COVID-19 transmit via expiratory particles?

Aerosol Sci Tech, 4/3/2020

Sima Asadi, Nicole Bouvier, Anthony S. Wexler & William D. Ristenpart

CDC reverses itself and says guidelines it posted on coronavirus airborne transmission were wrong

Agency removes statement, claiming website error *Washington Post*, 9/21/2020

Airborne transmission of SARS-CoV-2: The world should face the reality

Lidia Morawska^{a,*}, Junji Cao^b

Environment International, 4/10/2020

Coronavirus can be transmitted through the air, CDC confirms *Washington Post*, 10/5/2020

239 Experts With One Big Claim: The Coronavirus Is Airborne

The W.H.O. has resisted mounting evidence that viral particles floating indoors are infectious, some scientists say. The agency maintains the research is still inconclusive.

New York Times, 7/12/020

EDITORIAL | ONLINE FIRST

COVID-19 transmission—up in the air

The Lancet Respiratory Medicine

Published: October 29, 2020 • DOI: [https://doi.org/10.1016/S2213-2600\(20\)30514-2](https://doi.org/10.1016/S2213-2600(20)30514-2)

“Public health guidance now needs to advise people how to navigate risk in indoor settings” ...

Existing Guidelines: Single-Variable Bounds

- **Minimum social distance:** 6 feet / 1.8m (CDC), 1.5m (NL), 1.0m (WHO)
- Maximum **indoor gathering size:** 6 persons (OR), 10 (MN), 25 (MA), 3 households (CA)
- Maximum **outdoor gathering size:** 25 persons (MN, MA), 10 (MIT)
- Minimum **ventilation rate:** 6 ACH (ASHRAE), 10 L/s/person (UK)
- Minimum **open window time:** 5 min. open after 20 min. closed (Germany)
- Maximum **contact time:** 15 minutes (CDC, “contact” < 6 feet)



Common sense:

Each bound becomes *unsafe or unnecessary*, as other parameters vary.

The variables are obviously coupled, but how?

- CA Thanksgiving: < 3 households & < 2 hours

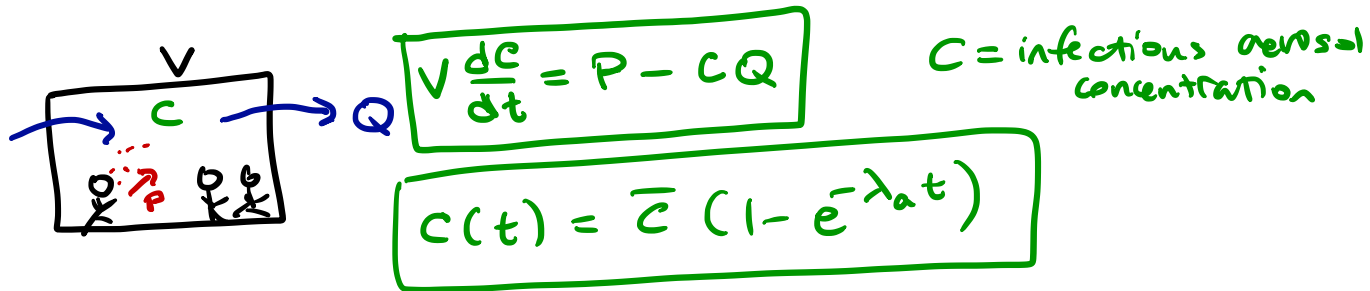
What if we wear masks, or test regularly?



COVID-19 Indoor Safety Guideline

Ref: MZ Bazant & JWM Bush, "Beyond Six Feet: A Guideline to Limit the Indoor Airborne Transmission of COVID-19", medRxiv preprint (2020)

Airborne Transmission in a Well-Mixed Room



$$\lambda_a = \frac{Q}{V} = \text{air exchange rate:}$$

$$\bar{C} = \frac{P}{Q} = \frac{P}{\lambda_a V} = \text{steady conc.}$$

$$P = \text{infectious aerosol production rate} = Q_b C_g P_m$$

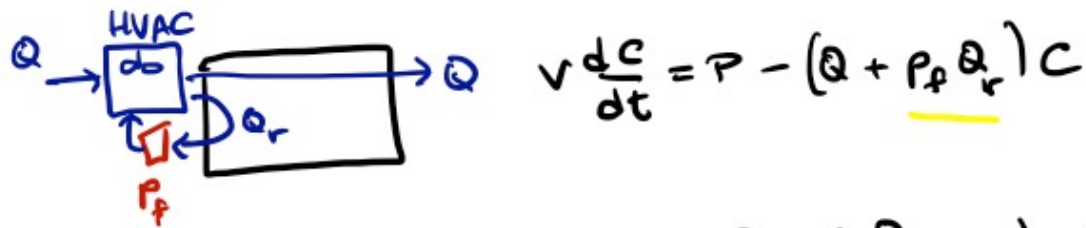
breathing flow rate
"infectiousness" of exhaled air
mask penetration factor

Transmission rate

$$\beta(t) = Q_b C(t) P_m \xrightarrow{t \gg 1}$$

$$\bar{\beta} = \frac{Q_b^2 C_g P_m^2}{\lambda_a V}$$

Air Filtration vs. Masks



Same as before with $\frac{Q}{V} \rightarrow \frac{Q + P_f Q_r}{V} = \lambda_a + P_f \lambda_r = \lambda$

$$\therefore \bar{\beta} = \frac{Q_b^2 C_g P_m^2}{(\lambda_a + P_f \lambda_r) V}$$

① Masks Best $P_m \approx 0.01$
OK $P_m \approx 0.3$

$$\therefore \frac{\bar{\beta}(\text{masks})}{\bar{\beta}(\text{no masks})} = P_m^2 \sim 10^{-1} - 10^{-4}$$

② Filtration:

Best case:

$$\frac{\bar{\beta}(\text{filter})}{\bar{\beta}(\text{no filter})} = \frac{\lambda_a}{\lambda_a + \lambda_f} = Z \gtrsim 0.1$$

outdoor
air fraction

Masks \gg filters

Safety Guideline

Epidemiological models (SIR)

$$\frac{dS}{dt} = -\beta SI \quad \beta = \text{transmission rate}$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

$$\frac{dR}{dt} = \gamma I \quad \gamma^{-1} = \text{recovery time (or death)}$$



early times: $S \approx S(0)$

$$I \approx I_0 e^{(R_0 - 1)\gamma t}$$

$$R_0 = \frac{\beta S_0}{\gamma} = \text{"reproductive number"}$$

> 1 epidemic

Indoor Safety

$$R_{in} = S_0 \int_0^{\tau} \beta dt < \epsilon$$

= expected # of transmission

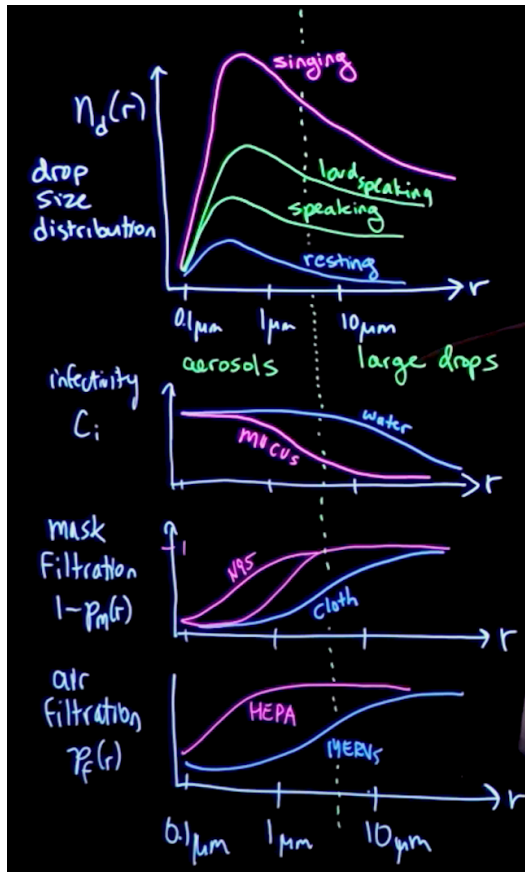
if $I_0 = 1, S_0 = N-1$

Steady:

$$(N-1)\bar{\beta}\tau < \epsilon$$

$$(N-1)\tau < \frac{\epsilon \lambda V}{Q_b^2 C_f P_m^2}$$

Airborne Transmission in a Well-Mixed Room



Mass balance for virion concentration / air volume (per radius)
 (Wells-Riley Model / Continuous Stirred Tank Reactor):

$$\frac{\partial C}{\partial t} = \frac{P(r)}{V} - \lambda_c(r)C$$

Virion production rate by exhaled drops / infected person

$$P(r) = Q_b c_v n_d(r) V_d(r) p_m(r)$$

mask transmission probability

Virion removal rate

$$\lambda_c(r) = \lambda_a \left(1 + (r/r_c)^2\right) + \lambda_f p_f(r) + \lambda_v(r)$$

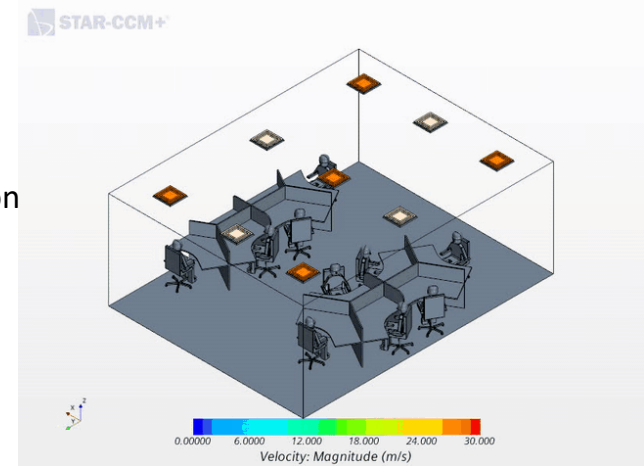
ventilation + sedimentation + filtration + deactivation

$$\lambda_a = \frac{Q}{V}, \quad \lambda_f = \frac{Q_f}{V}, \quad r_c = \sqrt{\frac{9\mu_a \lambda_a V}{2\rho_a g A}}$$

outdoor ACH, filtration ACH critical aerosol size

Transmission rate (exchange of "infection quanta")

$$\beta(t) = Q_b \int_0^\infty C(r,t) c_i(r) p_m(r) dr \rightarrow \frac{Q_b^2 p_m(\bar{r})^2 C_q}{\lambda_c(\bar{r}) V}$$



CFD simulation: Vinay Natrajan (Saint Gobain)

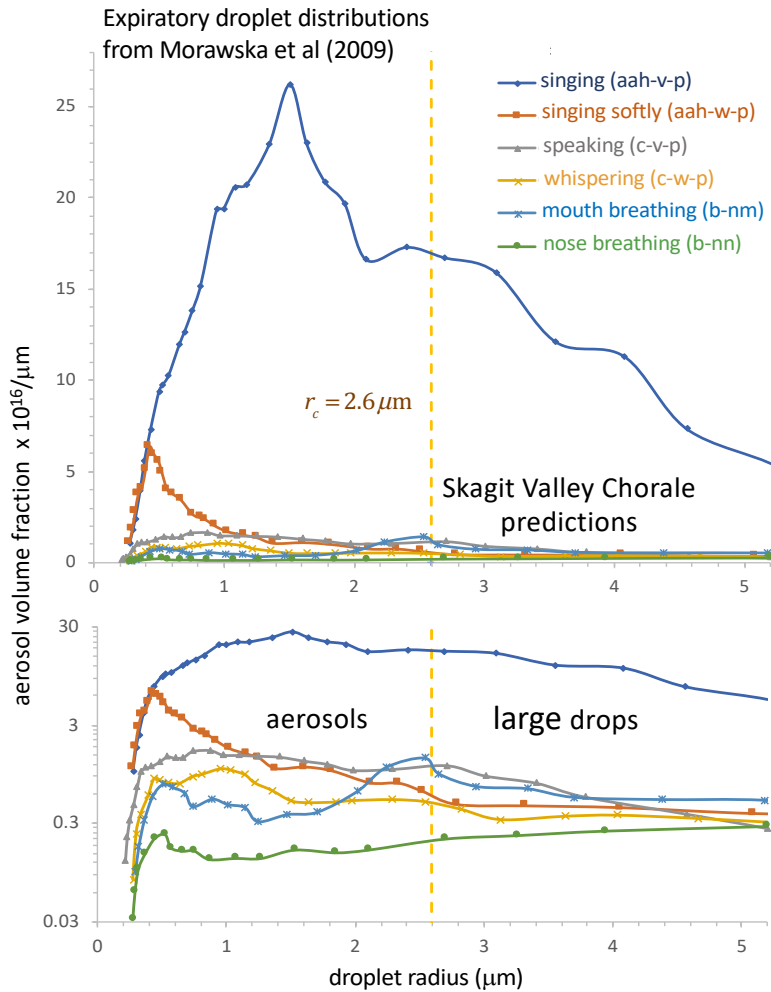
Infectiousness of exhaled air
 (quanta/volume)

$$C_q = \int_0^\infty c_v n_d(r) V_d(r) c_i(r) dr$$

Similar theory, spreadsheets:

- Lidia Morawska (Queensland)
- Jose-Luis Jimenez (U Colorado)

Inferring C_q from COVID-19 Super-Spreading Events



Exposed persons with slow incubation (Wells-Riley Model):

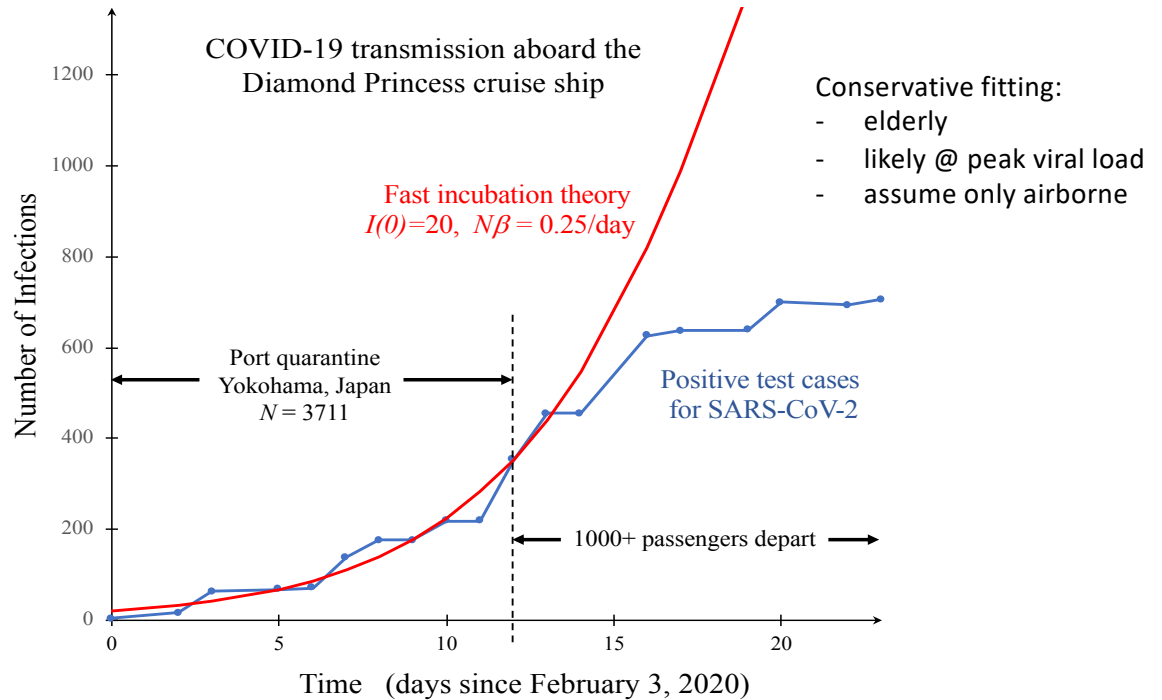
Skagit Valley Chorale, Ningbo bus,...

$$E(\tau) = S_0 \left(1 - \exp \left(- \int_0^\tau \beta(t) dt \right) \right)$$

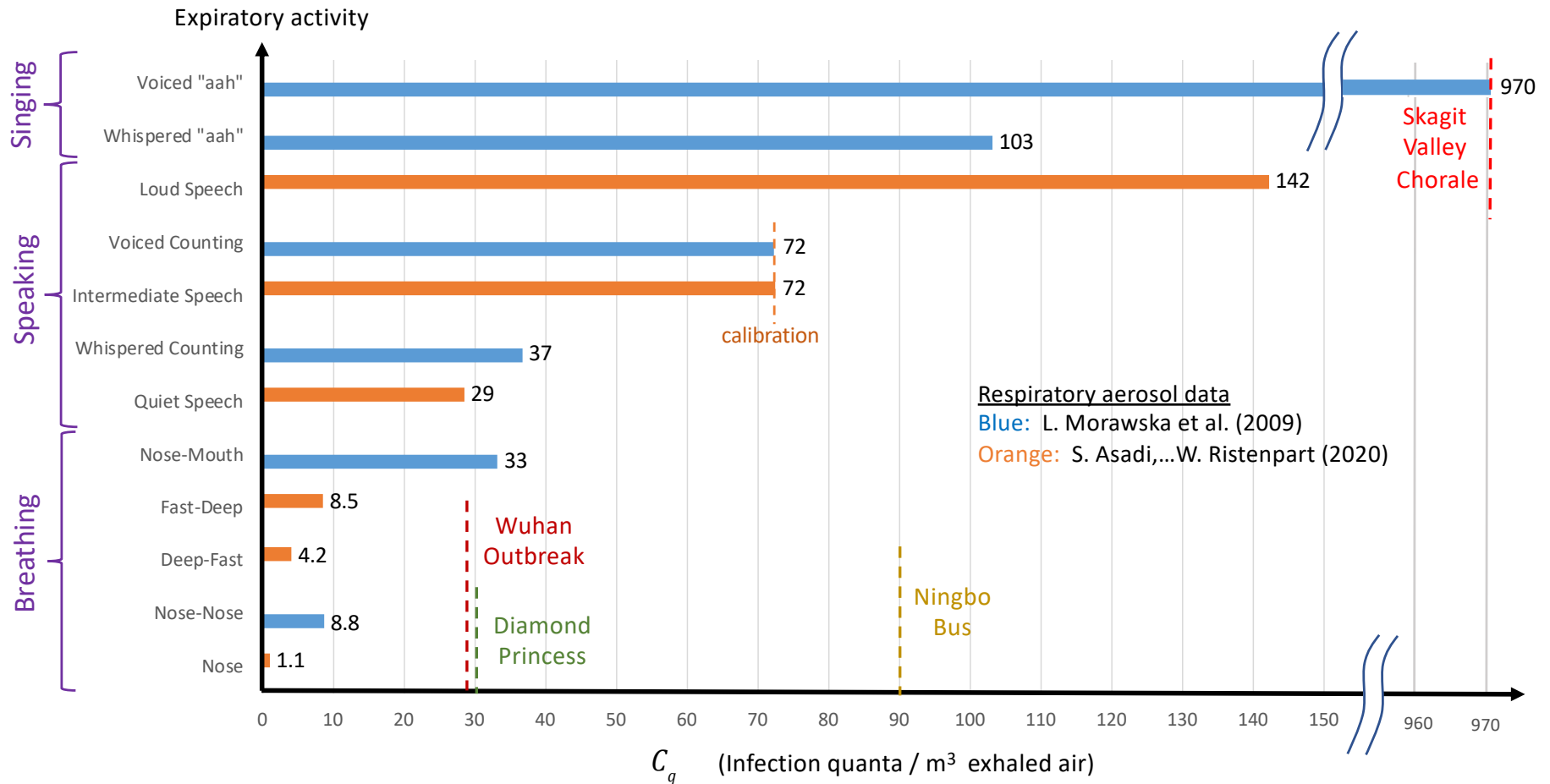
Infected persons with fast incubation:

Diamond Princess

$$I(\tau) = \left[N^{-1} - I_0^{-1} \left(\exp \left(N \int_0^\tau \beta(t) dt \right) - 1 \right) \right]^{-1}$$



Infectiousness of Exhaled Air vs. Activity



Also: SARS-CoV-2 infectious dose ~10 aerosolized virions, infectivity ~10% → 10x worse than SARS-Cov!

Universal Safety Guideline

- Epidemiological model: $S(t)$ =susceptible, $E(t)$ =exposed, $I(t)$ =infected

$$\frac{dS}{dt} = \beta(t)SI, \quad \frac{dE}{dt} = \beta(t)SI - \alpha E, \quad \frac{dI}{dt} = \alpha E, \quad S + E + I = N$$

- **Small tolerance for the “indoor reproductive number”** = expected number of transmissions *per* infected person entering the room for a given time

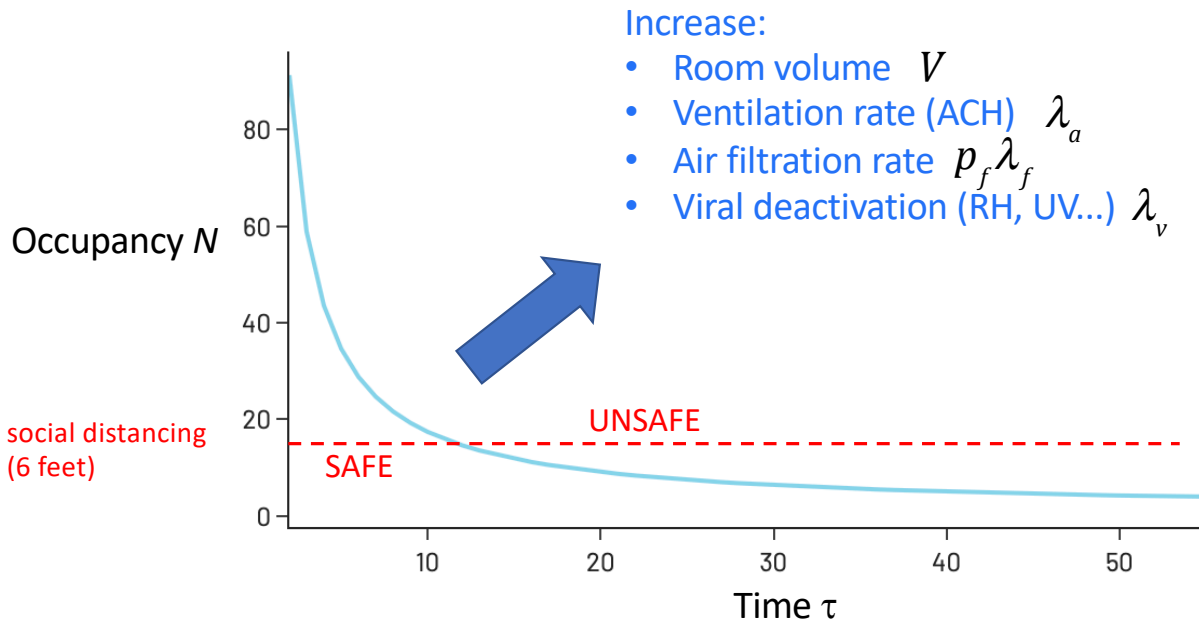
$$R_{in}(\tau) = E(\tau) + I(\tau) \sim S(0) \int_0^\tau \beta(t) dt = (N-1) \langle \beta \rangle \tau < \epsilon \ll 1$$

- Limits the **“cumulative exposure time”**:

$$(N-1) \tau < \frac{\epsilon \lambda_c(\bar{r}) V}{Q_b^2 p_m(\bar{r})^2 C_q}$$

COVID-19 Indoor Safety Guideline for Airborne Transmission

$$(N-1) \tau < \epsilon \frac{(\lambda_a(1+(\bar{r}/r_c)^2) + p_f(\bar{r})\lambda_f + \lambda_v(\bar{r})) V}{Q_b^2 p_m(\bar{r})^2 C_q s_r}$$



Increase:

- Room volume V
- Ventilation rate (ACH) λ_a
- Air filtration rate $p_f \lambda_f$
- Viral deactivation (RH, UV...) λ_v

Decrease:

- Mask penetration p_m
- Breathing rate Q_b
- Air infectiousness C_q
Singing > loud > soft speech > mouth > nose breathing
- Relative susceptibility s_r
Elderly (1.0) > adults (0.7) > children (25%)

Most effective (in order):

1. Wear masks! (~100x)
2. Increase ventilation (~10x)
3. Reduce exertion, speak softly (~10x)
4. Filter air. (~5x)
5. Maintain moderate RH

Spreadsheet

Online App

Safety Guideline for Indoor Airborne Transmission of COVID-19

Martin Z. Bazant

Contact: bazant@mit.edu <http://www.mit.edu/~bazant>
 History: 6-4-2020 (v1), 7-1-2020 (v2) <http://www.mit.edu/~bazant>
 This version: 10-8-2020 (v4.2) [medRxiv Version: 8-16-2020 \(v3\)](https://www.medrxiv.org/content/10.1101/2020.08.26.20182824v1)
 Reference: Martin Z. Bazant and John W. M. Bush, medRxiv preprint (2020)
 "Beyond Six Feet: A Guideline to Limit Indoor Airborne Transmission of COVID-19"
<https://www.medrxiv.org/content/10.1101/2020.08.26.20182824v1>

Input values in the pink cells. (Detailed instructions are in the next sheet.)

Physical Parameters

Floor area, A	870 ft ²	80.826 m ²
Mean ceiling height, H	10 ft	3.048 m
Room volume, V	8700 ft ³	246.36 m ³
Outdoor air change rate, λ_o	0.45 /hr (ACH)	
Ventilation (outdoor air) flow rate, Q	65.25 ft ³ /min	110.86 m ³ /hr
Primary outdoor air fraction, Z_p	0.15 (=1.0 natural ventilation)	
Return (recirculation) flow rate, Q_r	369.8 ft ³ /min	628.21 m ³ /hr
Primary (total) air flow rate, $Q+Q_r$	435 ft ³ /min	739.07 m ³ /hr
Total air change rate $(Q+Q_r)/V$	3 /h (ACH)	
Mean air velocity, $(Q+Q_r)/A$	0.5 ft/min	0.1524 m/min
Aerosol filtration efficiency, ρ_f	0.9 (>0.9997 HEPA, =0.2-0.9 MERVs, =0 no filter)	
Air filtration rate, λ_f	2.295 /hr	

Physiological Parameters

Mean breathing flow rate, Q_b	0.294 ft ³ /min	0.5 m ³ /hr (=0.5 rest, =1-3 active)
Respiratory aerosol radius, ζ	2 μm	(depends weakly on activity, disease)

Disease Parameters

Infectiousness of exhaled air, C_i	30 infection quanta/m ³	(depends on activity, Fig. 2)
Viral deactivation rate, λ_v	0.3 /hr	3.3333 hour deactivation time (can increase with UV and chemical disinfectants)

Infectious Aerosol Properties

Effective settling speed, $v_s(\zeta)$	0.48 mm/sec	1.728 m/hr.
Concentration relaxation rate, λ_c	3.612 /hr	0.2769 hour relaxation time
Dilution factor, f_d	6E-04 infectiousness of ambient air / exhaled breath	
Infectiousness of room air, $f_r C_i$	0.017 infection quanta/m ³	in steady state

Precautionary Parameters

Mask aerosol passage probability, p_m	1	(=1 no masks, ~0.1 cloth, <0.05 N95 surgical mask)
Airborne transmission rate, β_a	0.008 /hr	(per pair of persons in steady state)
Risk tolerance, ϵ	0.1	(bound on R_{top} , expected transmissions per infector)

Safe Room Occupancy

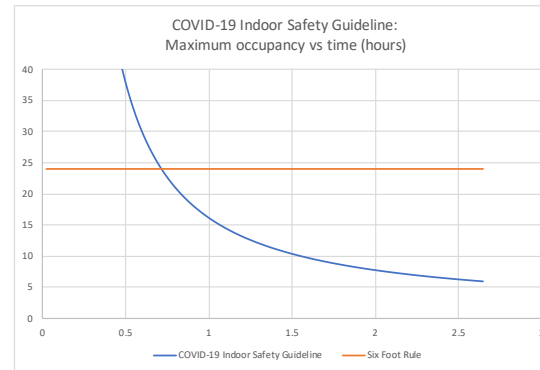
Exposure time, τ	2 hours	(net before testing/removal/recovery)
Maximum occupancy, N_{max}	7 persons	(with transient aerosol buildup)
	6 persons	(steady state aerosol concentration)

SIX FOOT RULE

Maximum occupancy, N	24 persons	
Minimum outdoor airflow per person	4.399 L/s/person	(need >3.8 schools, retail, >10 gyms)

Safe Exposure Time

Room occupancy, N	24 persons	
Maximum exposure time, t_{max}	0.715 hours	42.927 minutes (transient)
	0.516 hours	30.95 minutes (steady state)



9:56 📶 🔋

indoor-covid-safety.herokuapp.com

COVID-19 Indoor Safety Guideline

[Kasim Khan, John W. M. Bush, and Martin Z. Bazant](#)
 Beyond Six Feet: A Guideline to Limit Indoor Airborne Transmission of COVID-19 (Bazant & Bush, 2020)
<http://web.mit.edu/bazant/www/COVID-19/>
<https://github.com/kawesomexhan/covid-indoor>

Language: English British

English
 Français
 한국어
 简体中文

About

Room Specifications

Human Behavior

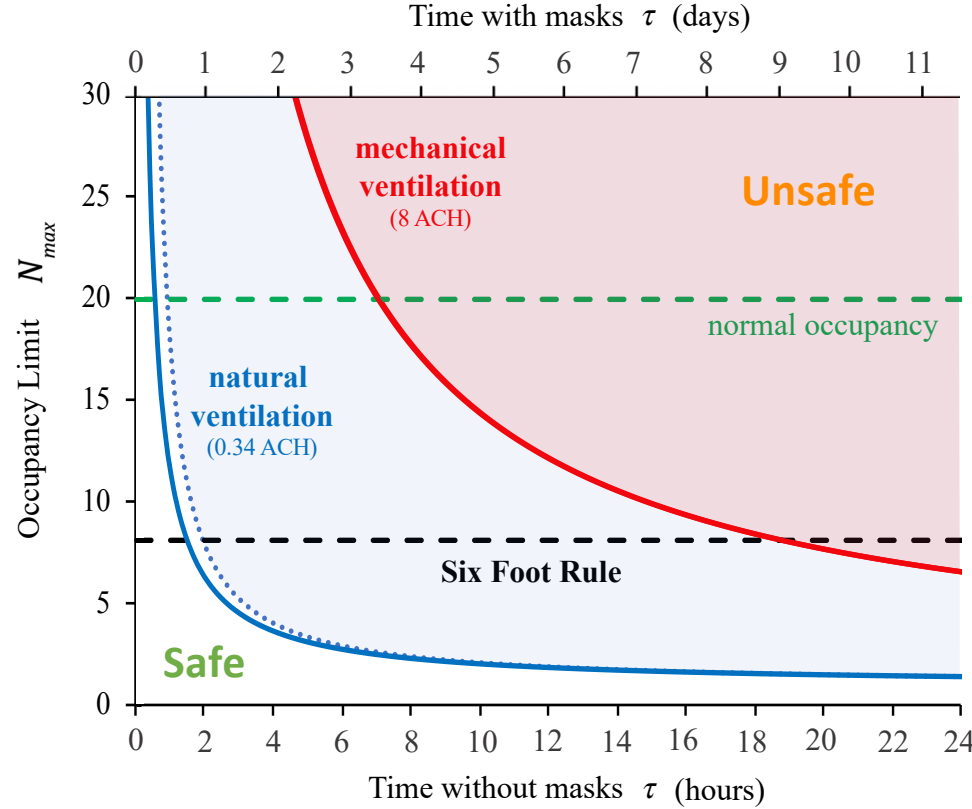
Frequently Asked Questions

About

To mitigate the spread of COVID-19, official public health guidelines have recommended limits on: person-to-person distance 16 feet /

Case Studies

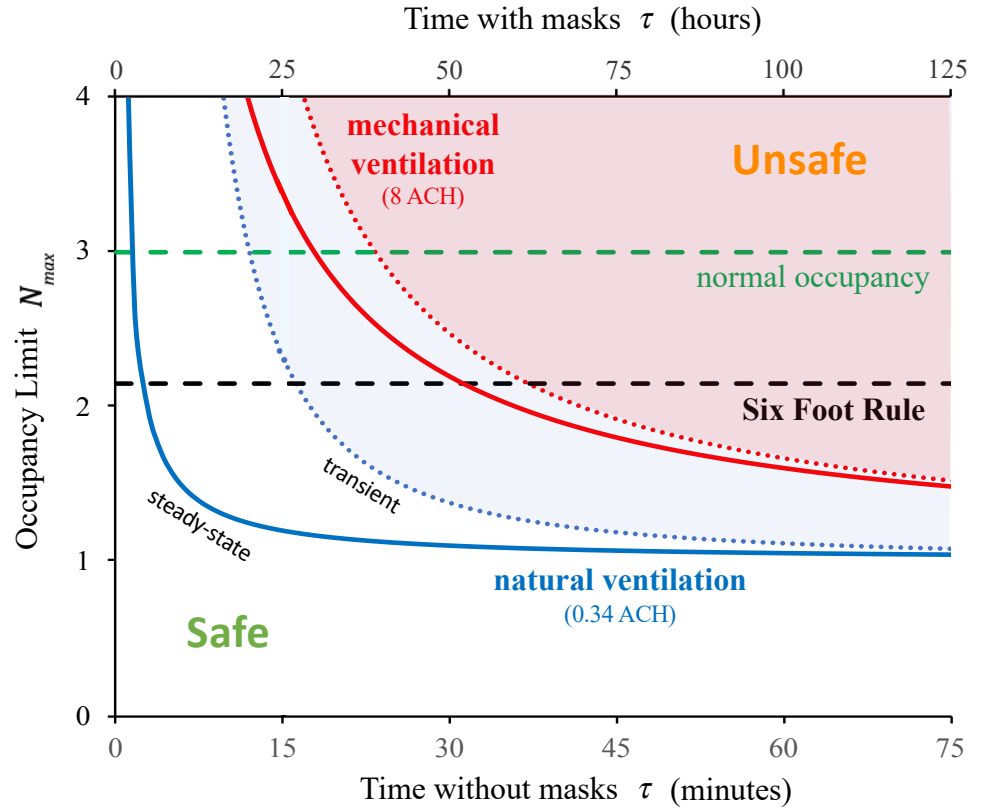
(a) Classroom



Low risk: $\epsilon = 10\%$ Children: $s_r = 25\%$ Cloth masks: $p_m = 30\%$

Validation: School transmissions are rare, more off campus.

(b) Nursing home

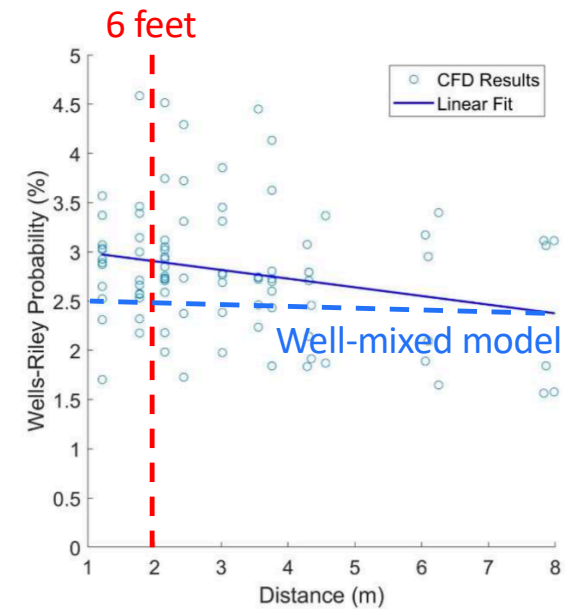
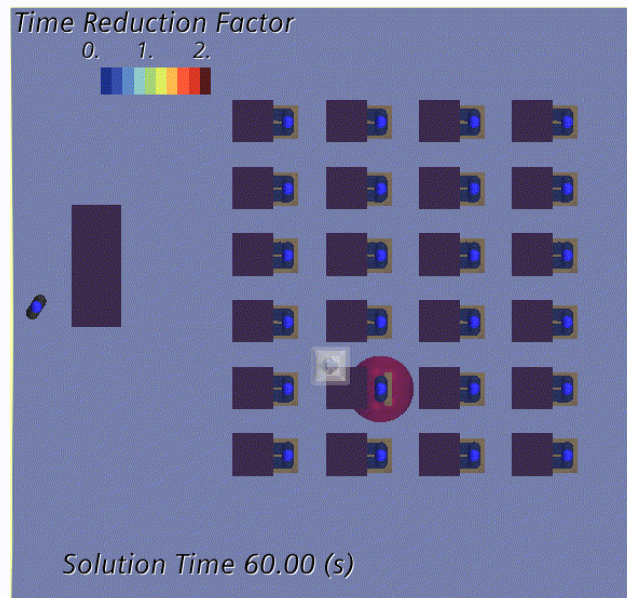
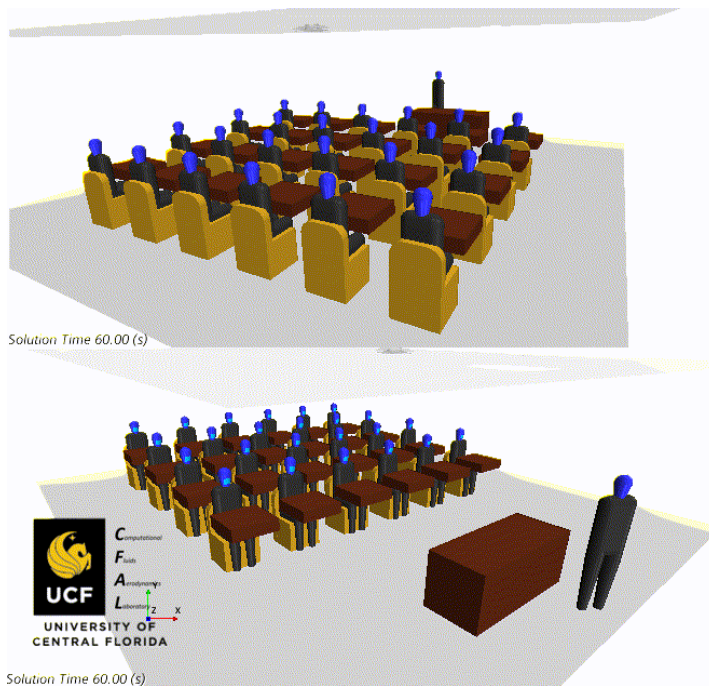


High risk: $\epsilon = 1\%$ Elderly: $s_r = 100\%$ Surgical masks: $p_m = 10\%$

Validation: 1/3 of global COVID-19 deaths in nursing homes

Beyond the Well Mixed Room

Natural Convection and Aerosol Mixing Indoors

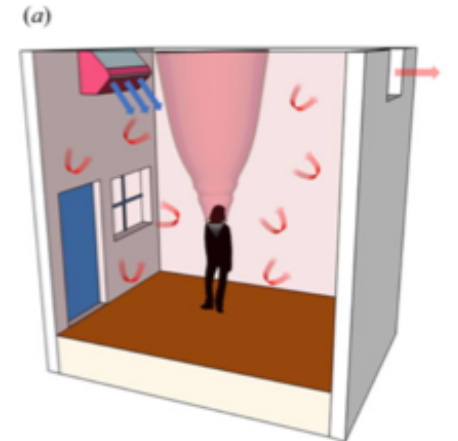
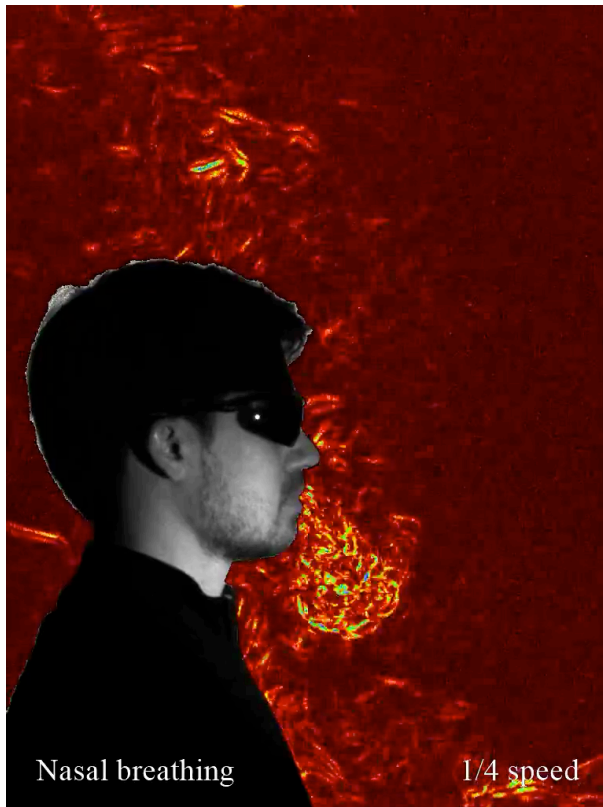


Foster & M. Kinzel,
Preprint (2021)

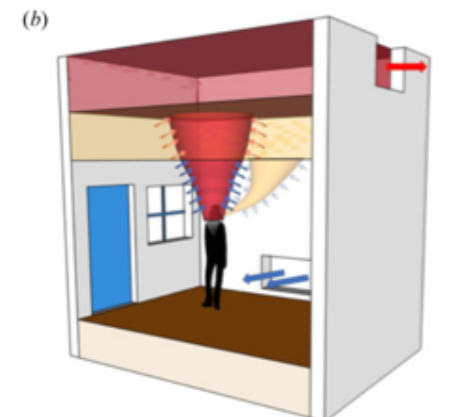
CFD simulation of a classroom (M. Kinzel, U. Central Florida)

Respiration, Ventilation & Natural Convection

Bhagat, Wykes, Dalziel & Linden, J. Fluid Mech. (2020)



Mixing Ventilation



Displacement Ventilation

Two Types of Airborne Transmission

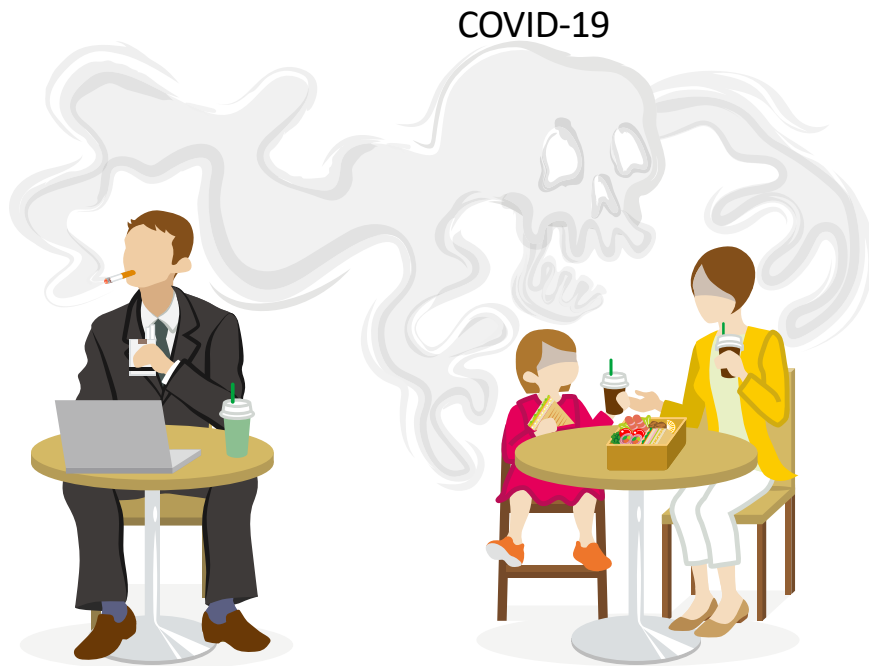


Short-range airborne transmission in turbulent plumes.

Long-range airborne transmission in a well-mixed room

<https://tinyurl.com/FAQ-aerosols>

We all understand airborne transmission.



Secondhand
smoke can infiltrate into
other units
through
hallways and
stairwells.

Don't be shy when it comes to your health. Talk to your building manager about making your apartment smokefree.



CDC.gov

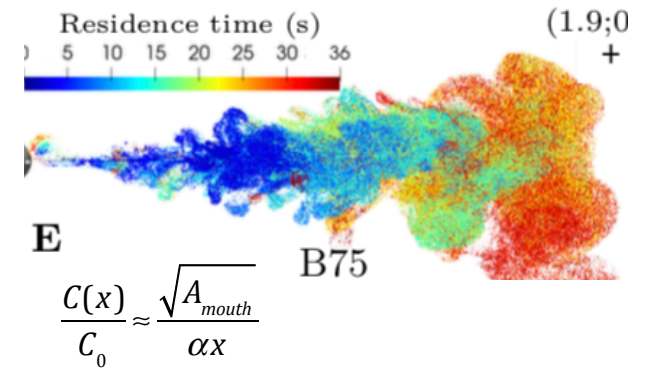
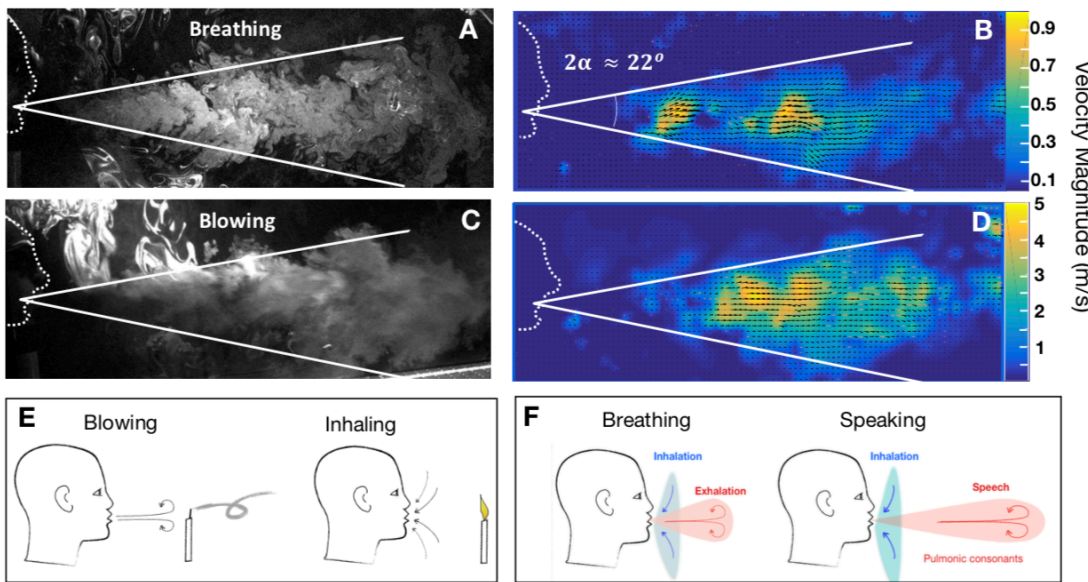
...and so can COVID-19 in a Korean apartment building!

Huang et al, Int. J. Infectious Diseases (2020)

Respiratory Turbulent Jets and Puff Trains

L. Bourouiba & J. W. M. Bush, *J. Fluid Mech.* (2014): Coughs and sneezes.

M. Abkarian, S. Mendez, N. Xue, F. Yang and H. A. Stone, *PNAS* (2020). Breathing and speaking.



$$R_{in}(\tau) \left(1 + \frac{\lambda_c(\bar{r}) V \sqrt{A_{mouth}} p_{jet}}{(N-1) Q_b \alpha x} \right) < \epsilon$$

Non-universal excess risk from short-range transmission

→ Social distancing, if masks are not worn

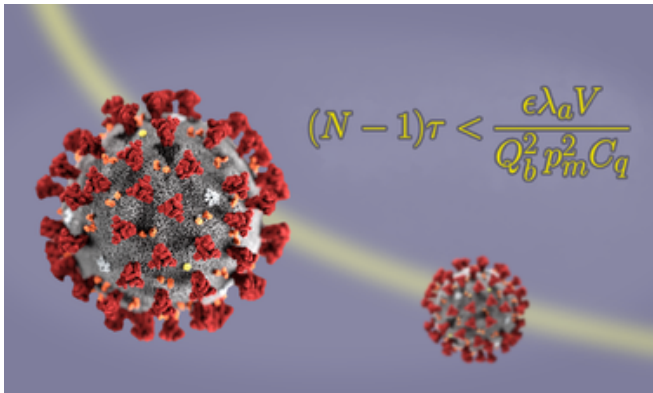
“Random packing”
of room occupants:

$$p_{jet} \approx (\tan^{-1} \alpha) / \pi$$

$$x \approx \sqrt{A/N}$$

COVID-19 Indoor Safety Guideline

Online app, spreadsheet: <http://www.mit.edu/~bazant/COVID-19>



- MOOC on edX: **10.S95x Physics of COVID-19 Transmission**
(free, self-paced) **Enroll now!**

- Assignment: MOOC Ch. 4 Homework (turn in written solution)

http://web.mit.edu/bazant/www/COVID-19/
<https://github.com/kawesomekhan/covid-indoor>

Language: English (selected), Français, 한국어, 简体中文

Units: British

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