IAP Mathematics Lecture, 1/25/2021, MIT

Mathematics of COVID-19 Transmission

Martin Z. Bazant

E. G. Roos (1944) Professor of Chemical Engineering
Professor of Mathematics
Massachusetts Institute of Technology

How does COVID-19 spread?

Modes of Transmission of Respiratory Pathogens

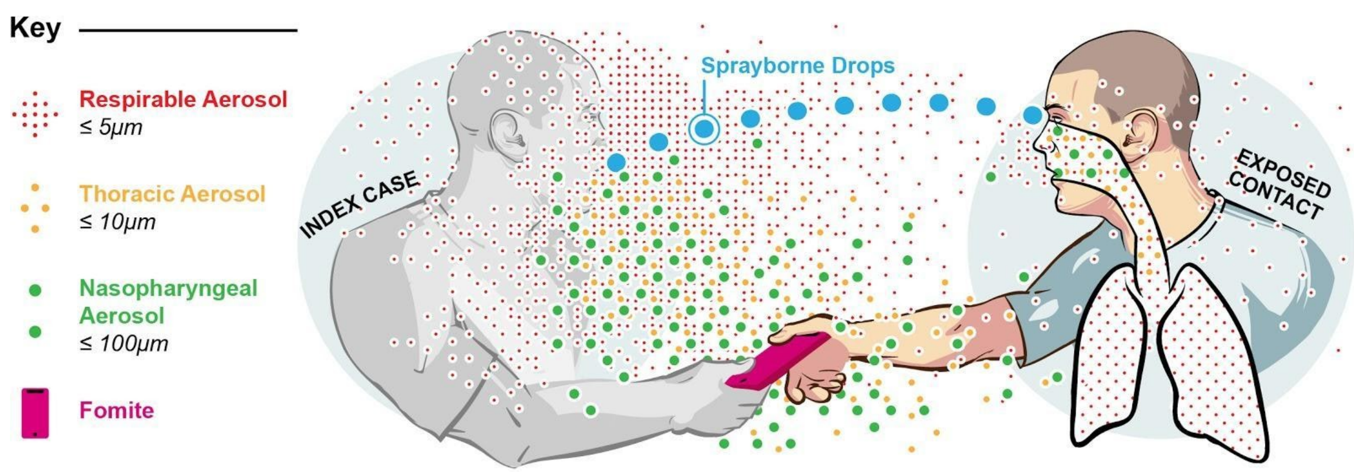
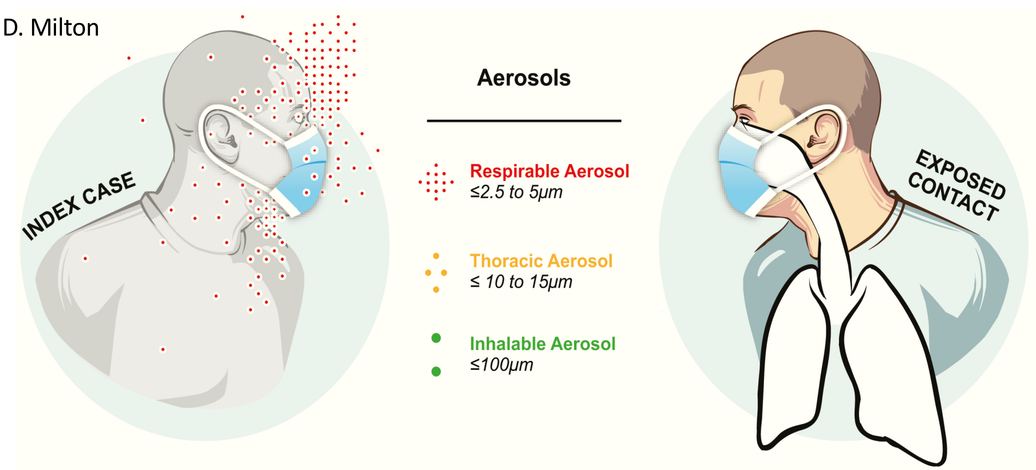


Image credit: D. Milton

1. Fomite / Contact
2. Large / Ballistic Drops
3. Airborne / Aerosol

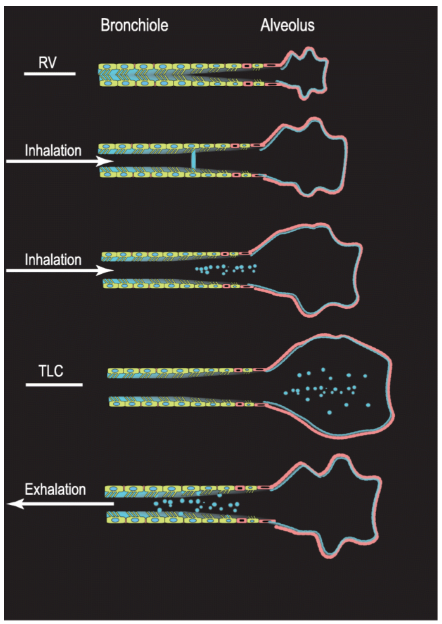


D. Milton, *J Pediatric Infect Dis Soc* (2020)

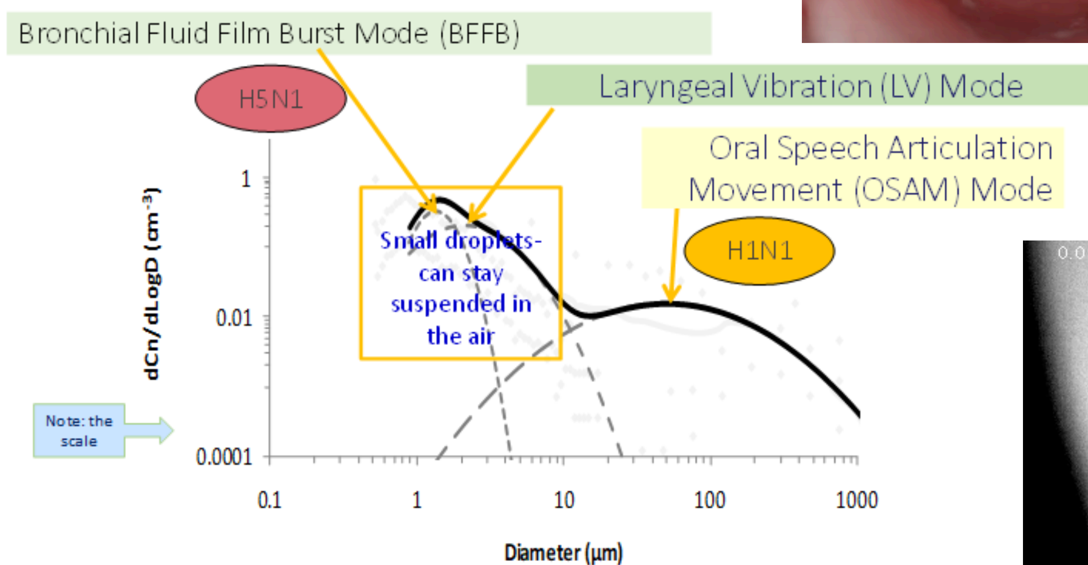
https://tinyurl.com/FAQ-aerosols

FAQs on Protecting Yourself from COVID-19 Aerosol Transmission
J. Jimenez, D. Milton, S. Miller, L. Marr, L. Morawska,…

Mechanisms for Expiratory Droplet Formation



B. Bake et al, Respiratory Research (2019)



Morawska et al., 2009. Size distribution and sites of origin of droplets expelled during expiratory activities. *Journal of Aerosol Science*, 40: 256-269, 2009.

Glottis (larynx) during speech
James P. Thomas, MD
voicedoctor.net

Abkarian & Stone, Phys. Rev. Fluids (2020)

Fate of Expiratory Drops

- *Pure liquid* drops settle or evaporate (Wells Curve):

Stokes settling time

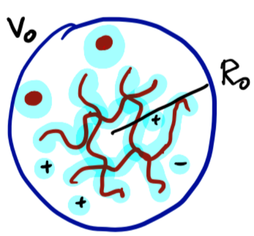
$$\tau_s = \frac{9\mu L}{2\rho g R^2}$$

Evaporation time

$$\tau_e = \frac{R_0^{\ 2}}{\bar{D}(1-RH)}$$

- "Droplet nuclei" are *stabilized by solutes* (mucins, salts…) and remain suspended

$$\frac{R_{eq}}{R_0} = \left(\frac{\phi^0_{solutes}}{1-RH}\right)^{1/3}$$



R. Netz, J. Phys. Chem. B (2020)

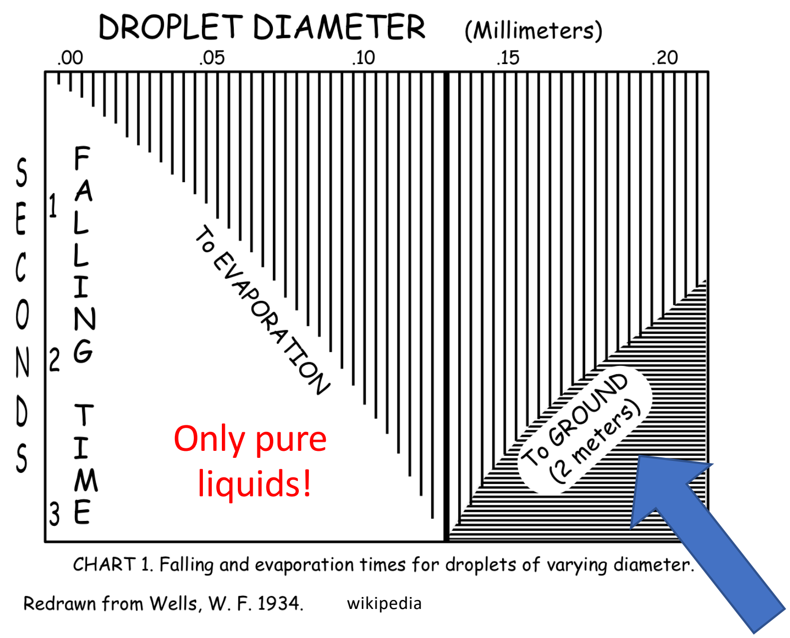


CHART 1. Falling and evaporation times for droplets of varying diameter.

Redrawn from Wells, W. F. 1934. wikipedia

(Origin of the 6 Foot Rule!)

Indoor Airborne Transmission: *Overwhelming Evidence*

- *All* 1200+ super-spreading events indoors, presumed airborne

  [SARS-CoV-2 Superspreading Event Database, https://covid19settings.blogspot.com/p/project.html]

  - Jan. 19, 2020: Tiangtong Temple, Ningbo, China. 2 buses 23/68 infected by 1 person on one bus (100min ride), none on the other bus; no correlation with seating
  - Feb. 3-15, 2020: Diamond Princess cruise ship. 12-day quarantine in Yokohama. ~20 cases →354/3711 infected on board; no correlation with shared rooms
  - March 10, 2020: Skagit Valley Chorale, WA. 53/61 infected by one in 2.5-hour choir practice (2 died, median age 69). no correlation with "close contacts"

- Indoor transmission from Wuhan across China

  [Qian et. al, medRxiv (2020).]

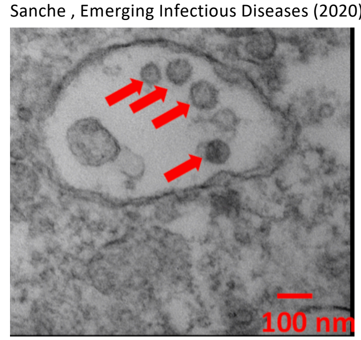
  - 7324 initial cases in 320 cities outside Hubei Province
  - *All* 318 outbreaks occurred indoors (2 or more transmissions)
  - 80% in family apartments, 34% involved public transportation
  - Only 1 cluster outdoors (2 transmissions)

Sanche , Emerging Infectious Diseases (2020)

- Patient-generated infectious aerosols confirmed

  [Santarpia et. al, medRxiv (2020). Lednicky et al, Int. J. Infectious Diseases (2020)]

  - Collected in hospital rooms 16 feet from patients
  - SARS-CoV-2 virions present and most infectious in <4um droplets
  - Similar to SARS coronavirus, measles, tuberculosis,…



Santarpia et al, medRxiv (2020)

Aerosol Science vs. Public Health Guidance

The coronavirus pandemic and aerosols: Does COVID-19 transmit via expiratory particles?

Aerosol Sci Tech, 4/3/2020

Sima Asadi, Nicole Bouvier, Anthony S. Wexler & William D. Ristenpart

CDC reverses itself and says guidelines it posted on coronavirus airborne transmission were wrong

Agency removes statement, claiming website error *Washington Post, 9/21/2020*

Airborne transmission of SARS-CoV-2: The world should face the reality

Lidia Morawska[a,*], Junji Cao[b]

Environment International, 4/10/2020

Coronavirus can be transmitted through the air, CDC confirms *Washington Post, 10/5/2020*

239 Experts With One Big Claim: The Coronavirus Is Airborne

The W.H.O. has resisted mounting evidence that viral particles floating indoors are infectious, some scientists say. The agency maintains the research is still inconclusive.

New York Times, 7/12/020

EDITORIAL | ONLINE FIRST

COVID-19 transmission—up in the air

The Lancet Respiratory Medicine

Published: October 29, 2020 • DOI: https://doi.org/10.1016/S2213-2600(20)30514-2

"Public health guidance now needs to advise people how to navigate risk in indoor settings"…

Existing Guidelines: Single-Variable Bounds

- **Minimum social distance: 6 feet** / 1.8m (CDC), 1.5m (NL), 1.0m (WHO)
- Maximum indoor gathering size: 6 persons (OR), 10 (MN), 25 (MA), 3 households (CA)
- Maximum outdoor gathering size: 25 persons (MN, MA), 10 (MIT)
- Minimum ventilation rate: 6 ACH (ASHRAE), 10 L/s/person (UK)
- Minimum open window time: 5 min. open after 20 min. closed (Germany)
- Maximum contact time: 15 minutes (CDC, “contact” < 6 feet)

Common sense:

Each bound becomes ***unsafe*** *or* ***unnecessary***,
as other parameters vary***.***

The variables are obviously coupled, but how?

- CA Thanksgiving: < 3 households & < 2 hours

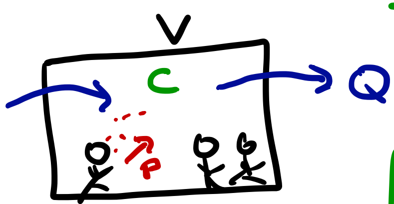
What if we wear masks, or test regularly?

COVID-19 Indoor Safety Guideline

Ref: MZ Bazant & JWM Bush, “Beyond Six Feet: A Guideline to Limit the Indoor Airborne Transmission of COVID-19”, medRxiv preprint (2020)

IAP Math Lecture 1/27/21

Airborne Transmission in a Well-Mixed Room



$$V\frac{dC}{dt} = P - CQ$$

C = infectious aerosol concentration

$$C(t) = \bar{C}\left(1 - e^{-\lambda_a t}\right)$$

$\lambda_a = \frac{Q}{V}$ = air exchange rate :

$\bar{C} = \frac{P}{Q} = \frac{P}{\lambda_a V}$ = steady conc.

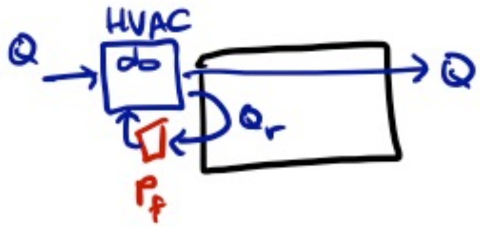
P = infectious aerosol production rate $= Q_b\, C_q\, p_m$

(Q_b: breathing flow rate; C_q: "infectiousness" of exhaled air; p_m: mask penetration factor)

Transmission rate

$$\beta(t) = Q_b\, C(t)\, p_m \xrightarrow{\lambda_a t \gg 1} \boxed{\bar{\beta} = \frac{Q_b^2 C_q p_m^2}{\lambda_a V}}$$

Air Filtration vs. Masks



$$V\frac{dc}{dt} = P - (Q + p_f Q_r)C$$

Same as before with $\frac{Q}{V} \to \frac{Q + p_f Q_r}{V} = \lambda_a + p_f \lambda_r = \lambda$

$$\therefore \boxed{\bar{\beta} = \frac{Q_b^2 C_q p_m^2}{(\lambda_a + p_f \lambda_f) V}}$$

① **Masks**

Best $p_m \simeq 0.01$

OK $p_m \simeq 0.3$

$$\therefore \frac{\bar{\beta}(\text{masks})}{\bar{\beta}(\text{no masks})} = p_m^2 \sim 10^{-1} - 10^{-4}$$

② **Filtration:**

Best case:

$$\frac{\bar{\beta}(\text{filter})}{\bar{\beta}(\text{no filter})} = \frac{\lambda_a}{\lambda_a + \lambda_f} = Z \gtrsim 0.1$$

Z = outdoor air fraction

Masks ≫ filters

Safety Guideline

Epidemiological models (SIR)

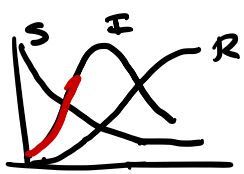
$$\frac{dS}{dt} = -\beta S I$$

β = transmission rate

$$\frac{dI}{dt} = \beta S I - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

γ^{-1} = recovery time (or death)



early times : $S \approx S(0)$

$$I \simeq I_0\, e^{(R_0 - 1)\gamma t}$$

$R_0 = \frac{\beta S_0}{\gamma}$ = "reproductive number"

> 1 epidemic

Indoor Safety

$$R_{in} = S_0 \int_0^{\tau} \beta\, dt < \epsilon$$

= expected # of transmission

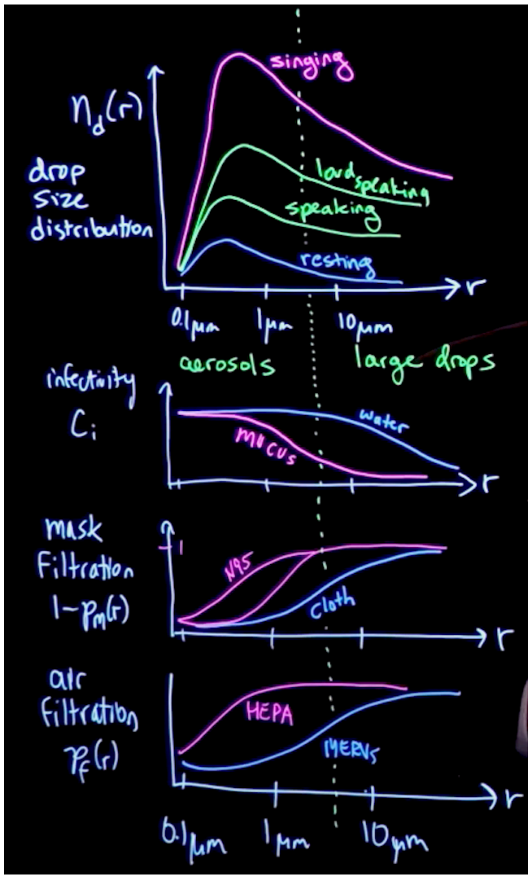
if $I_0 = 1$, $S_0 = N-1$

Steady:

$$(N-1)\bar{\beta}\tau < \epsilon$$

$$(N-1)\tau < \frac{\epsilon \lambda V}{Q_b^2 C_q P_m^2}$$

Airborne Transmission in a Well-Mixed Room



Mass balance for virion concentration / air volume (per radius)

(Wells-Riley Model / Continuous Stirred Tank Reactor):

$$\frac{\partial C}{\partial t} = \frac{P(r)}{V} - \lambda_c(r)C$$

Virion production rate by exhaled drops / infected person

$$P(r) = Q_b c_v n_d(r) V_d(r) p_m(r)$$

mask transmission probability

Virion removal rate

$$\lambda_c(r) = \lambda_a\left(1 + (r/r_c)^2\right) + \lambda_f p_f(r) + \lambda_v(r)$$

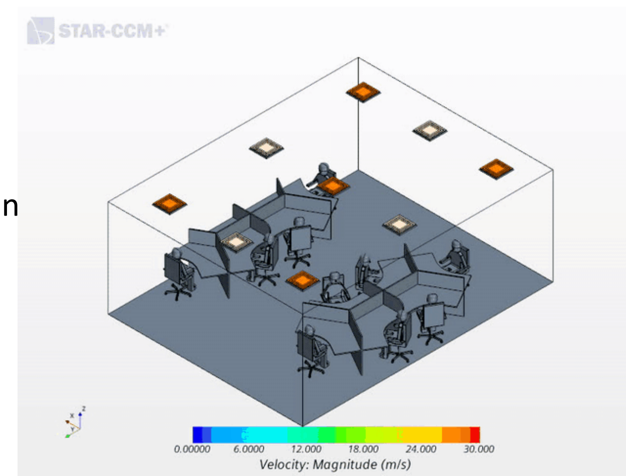
ventilation + sedimentation + filtration + deactivation

$$\lambda_a = \frac{Q}{V}, \quad \lambda_f = \frac{Q_f}{V} \qquad r_c = \sqrt{\frac{9\mu_a \lambda_a V}{2\rho_d g A}}$$

outdoor ACH , filtration ACH — critical aerosol size

Transmission rate (exchange of "infection quanta")

$$\beta(t) = Q_b \int_0^\infty C(r,t) c_i(r) p_m(r) dr \rightarrow \frac{Q_b^2 p_m(\bar{r})^2 C_q}{\lambda_c(\bar{r}) V}$$



CFD simulation: Vinay Natrajan (Saint Gobain)

Infectiousness of exhaled air (quanta/volume)

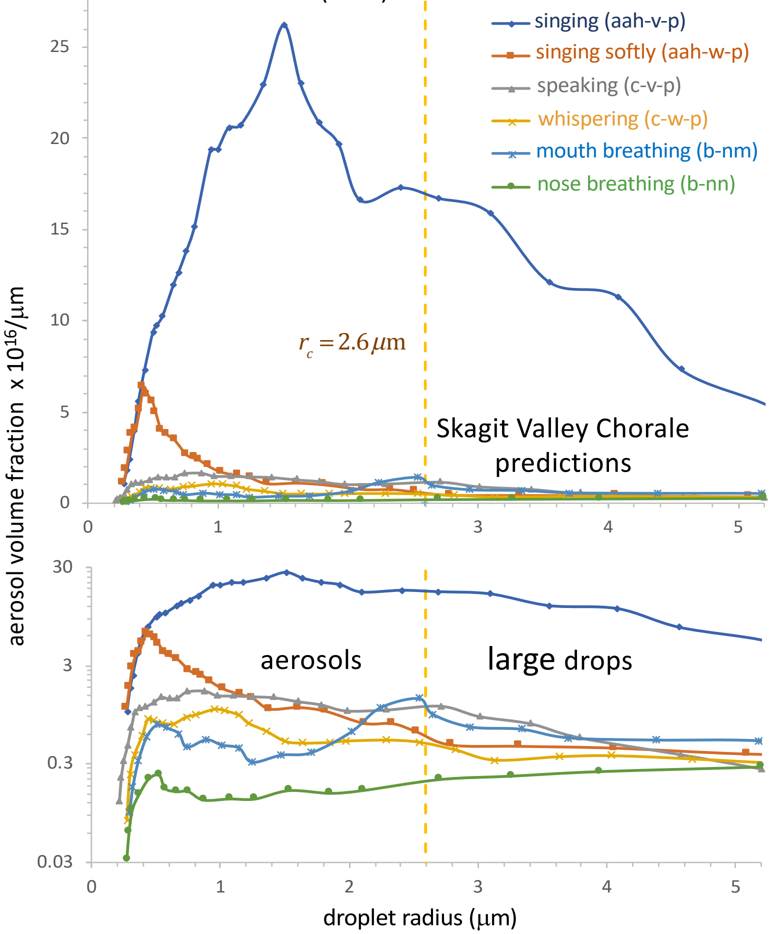
$$C_q = \int_0^\infty c_v n_d(r) V_d(r) c_i(r) dr$$

Similar theory, spreadsheets:

- Lidia Morawska (Queensland)
- Jose-Luis Jimenez (U Colorado)

Inferring C_q from COVID-19 Super-Spreading Events

Expiratory droplet distributions
from Morawska et al (2009)

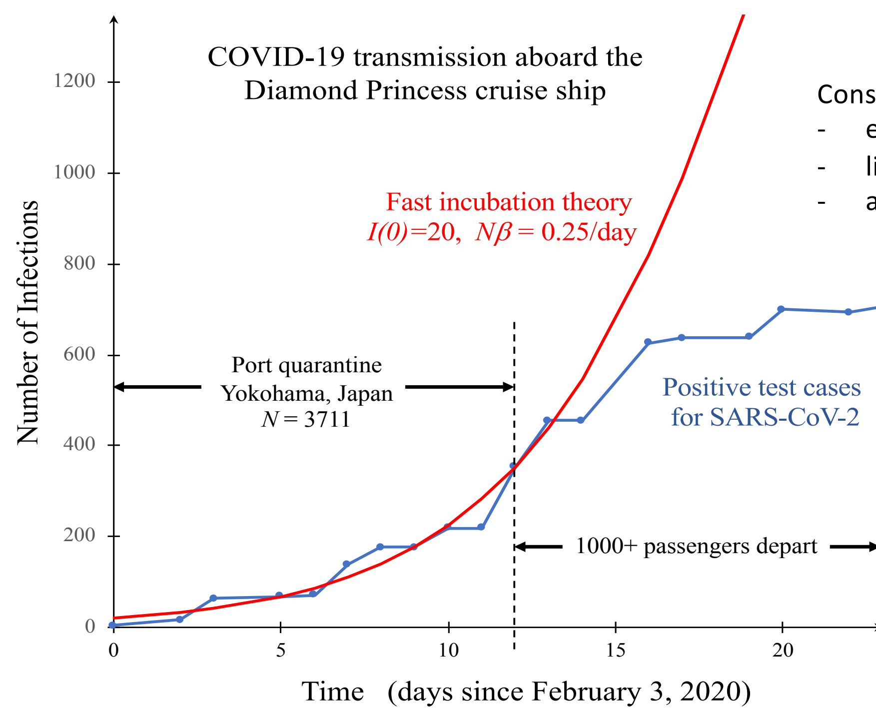


Exposed persons with slow incubation (Wells-Riley Model):
Skagit Valley Chorale, Ningbo bus,…

$$E(\tau) = S_0\left(1 - \exp\left(-\int_0^\tau \beta(t)dt\right)\right)$$

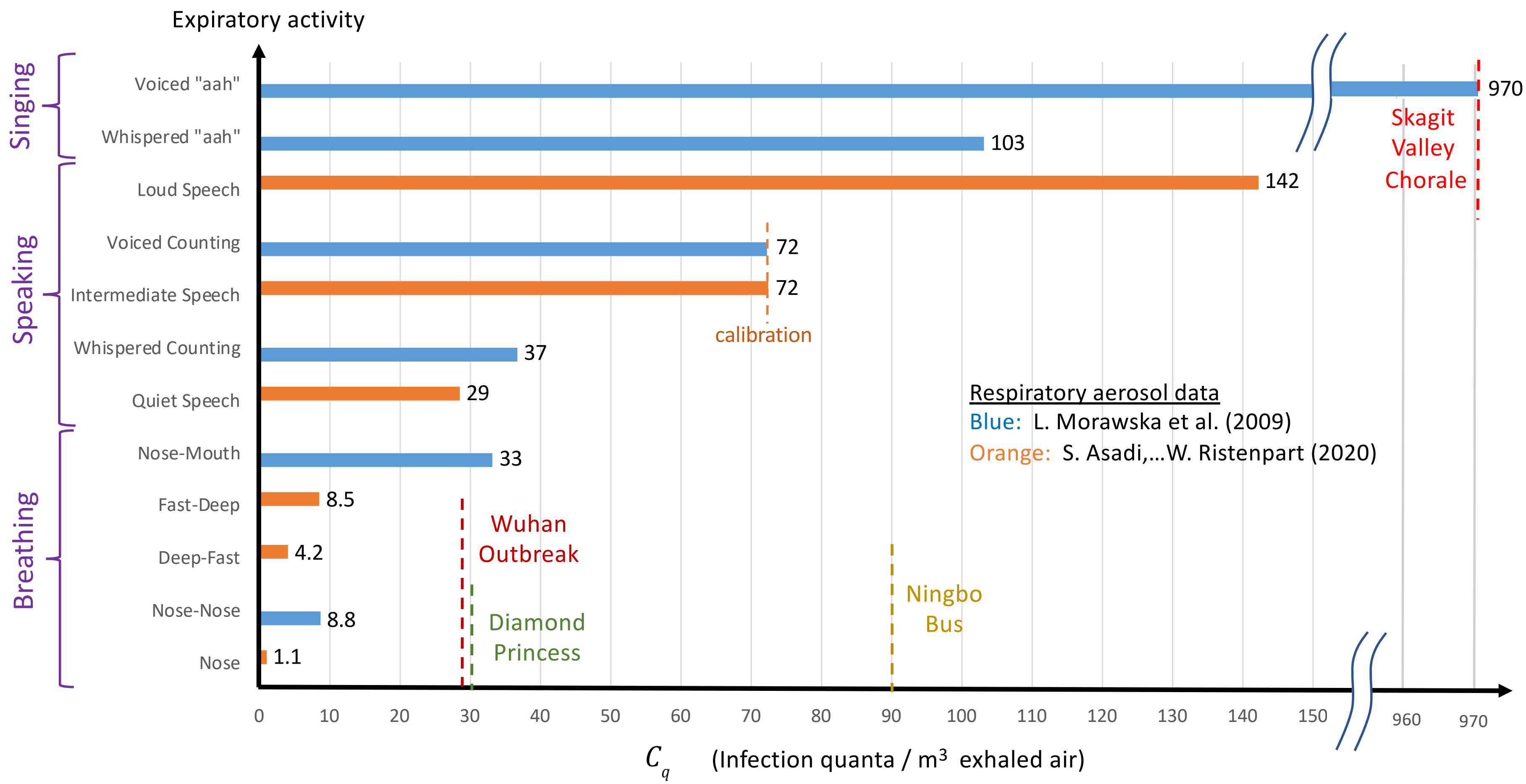
Infected persons with fast incubation:
Diamond Princess

$$I(\tau) = \left[N^{-1} - I_0^{-1}\left(\exp\left(N\int_0^\tau \beta(t)dt\right) - 1\right)^{-1}\right]^{-1}$$



Conservative fitting:
- elderly
- likely @ peak viral load
- assume only airborne

Infectiousness of Exhaled Air vs. Activity



Also: SARS-CoV-2 infectious dose ~10 aerosolized virions, infectivity ~10% → 10x worse than SARS-Cov!

Universal Safety Guideline

- Epidemiological model: S(t)=susceptible, E(t)=exposed, I(t)=infected

$$\frac{dS}{dt} = \beta(t)SI, \quad \frac{dE}{dt} = \beta(t)SI - \alpha E, \quad \frac{dI}{dt} = \alpha E, \quad S + E + I = N$$

- Small tolerance for the "indoor reproductive number" = expected number of transmissions *per* infected person entering the room for a given time

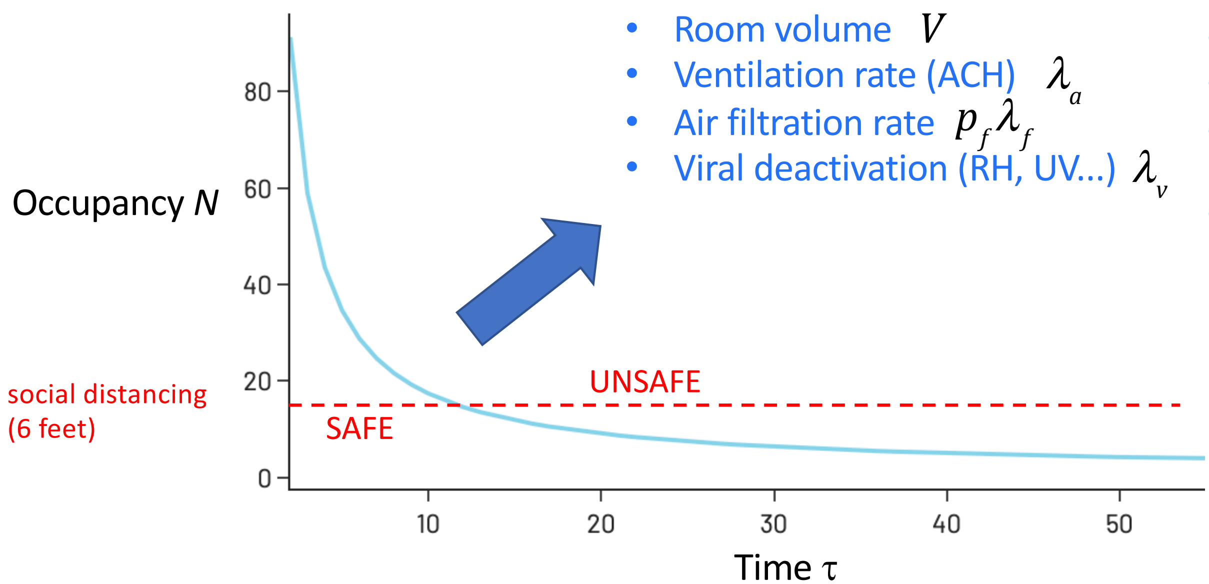
$$R_{in}(\tau) = E(\tau) + I(\tau) \sim S(0)\int_0^\tau \beta(t)dt = (N-1)\langle\beta\rangle\tau < \epsilon \ll 1$$

- Limits the "cumulative exposure time":

$$(N-1)\,\tau < \frac{\epsilon\,\lambda_c(\bar{r})\,V}{Q_b^2\,p_m(\bar{r})^2\,C_q}$$

COVID-19 Indoor Safety Guideline for Airborne Transmission

$$(N-1)\,\tau < \epsilon\,\frac{\left(\lambda_a\left(1+(\bar{r}/r_c)^2\right)+p_f(\bar{r})\lambda_f+\lambda_v(\bar{r})\right)V}{Q_b^2\,p_m(\bar{r})^2\,C_q s_r}$$



Increase:

- Room volume V
- Ventilation rate (ACH) λ_a
- Air filtration rate $p_f\lambda_f$
- Viral deactivation (RH, UV...) λ_v

Decrease:

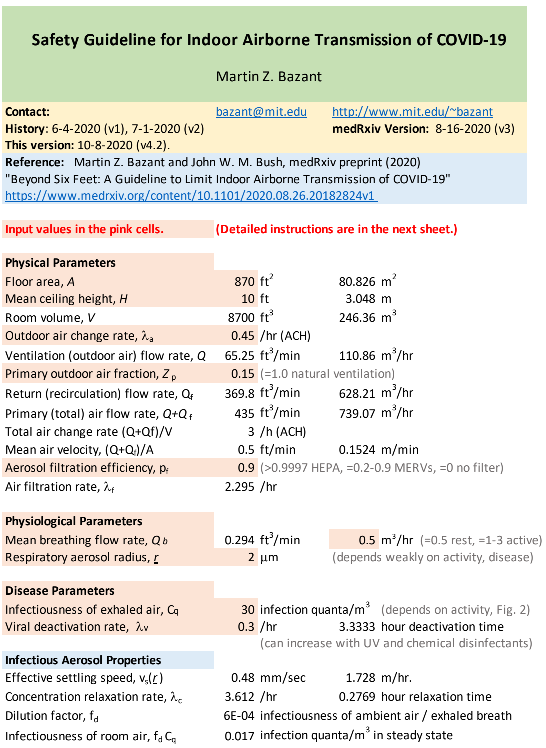
- Mask penetration p_m
- Breathing rate Q_b
- Air infectiousness C_q
  Singing > loud > soft speech > mouth > nose breathing
- Relative susceptibility s_r
  Elderly (1.0) > adults (0.7) > children (25%)

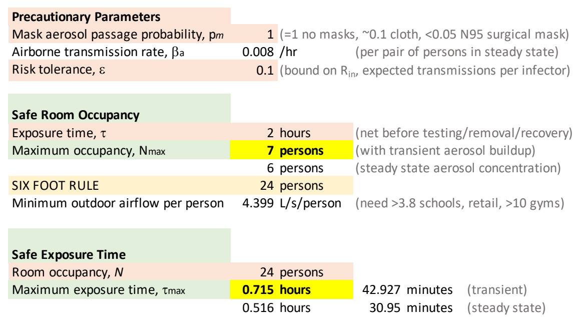
Most effective (in order):

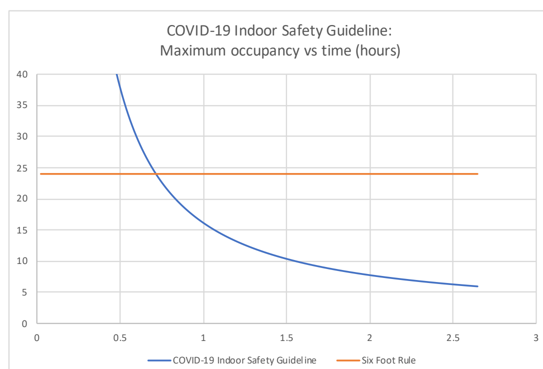
1. Wear masks! (~100x)
2. Increase ventilation (~10x)
3. Reduce exertion, speak softly (~10x)
4. Filter air. (~5x)
5. Maintain moderate RH

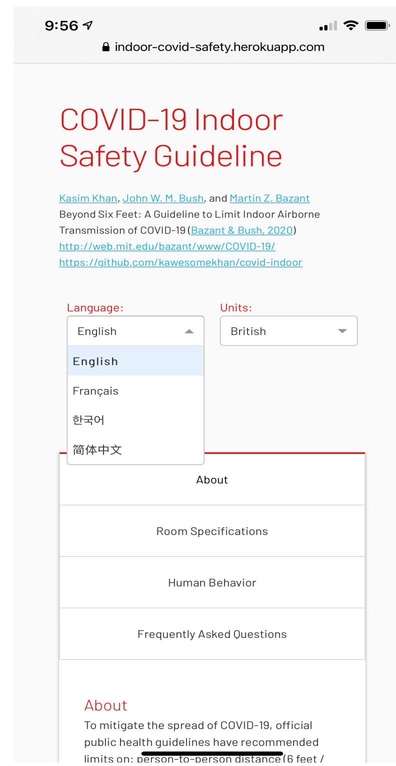
Spreadsheet

Online App

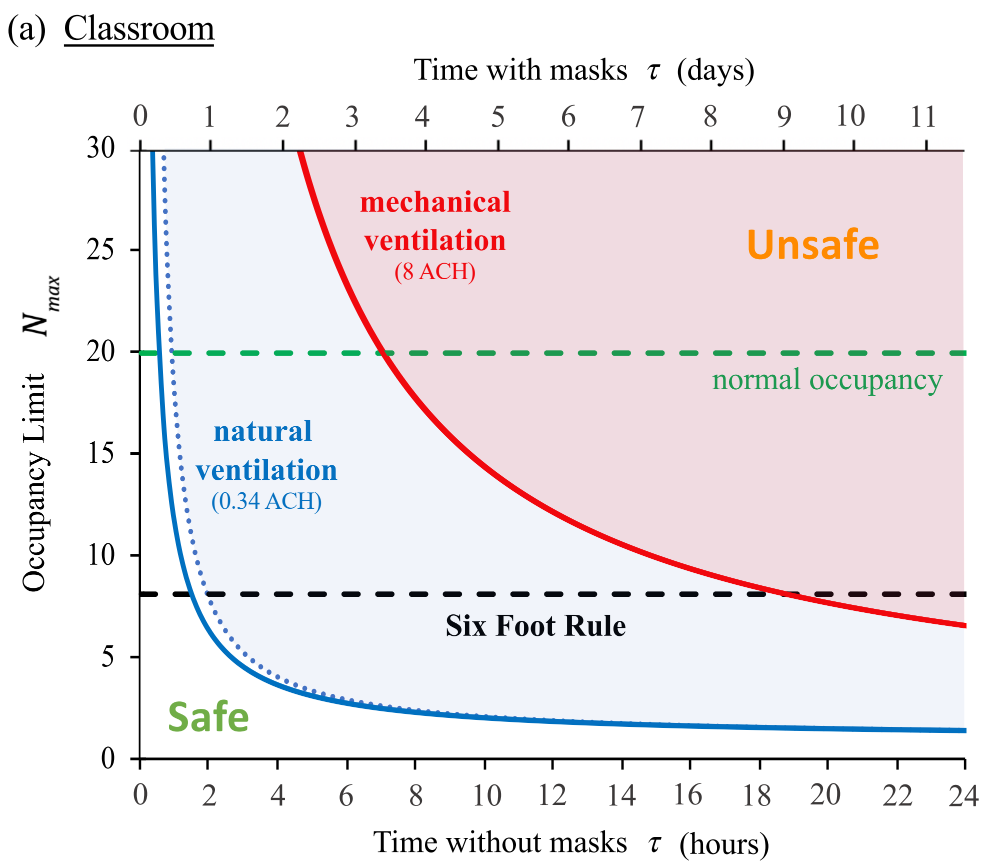






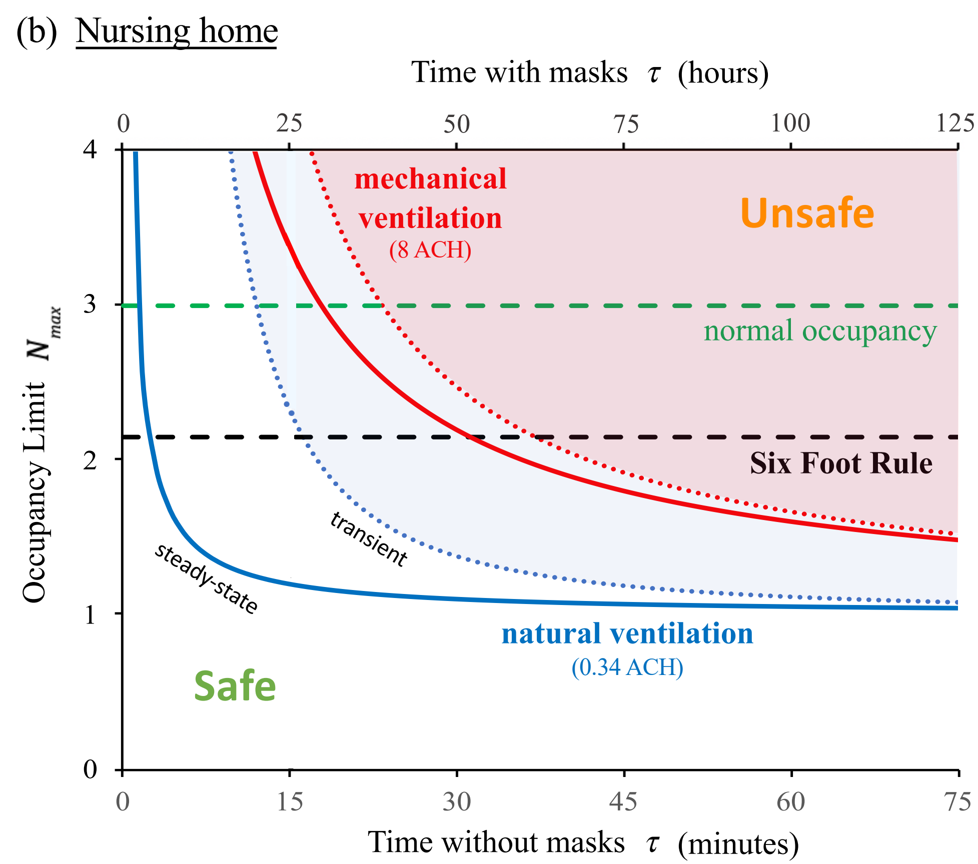


Case Studies



Low risk: $\epsilon = 10\%$ Children: $s_r = 25\%$ Cloth masks: $p_m = 30\%$

Validation: School transmissions are rare, more off campus.

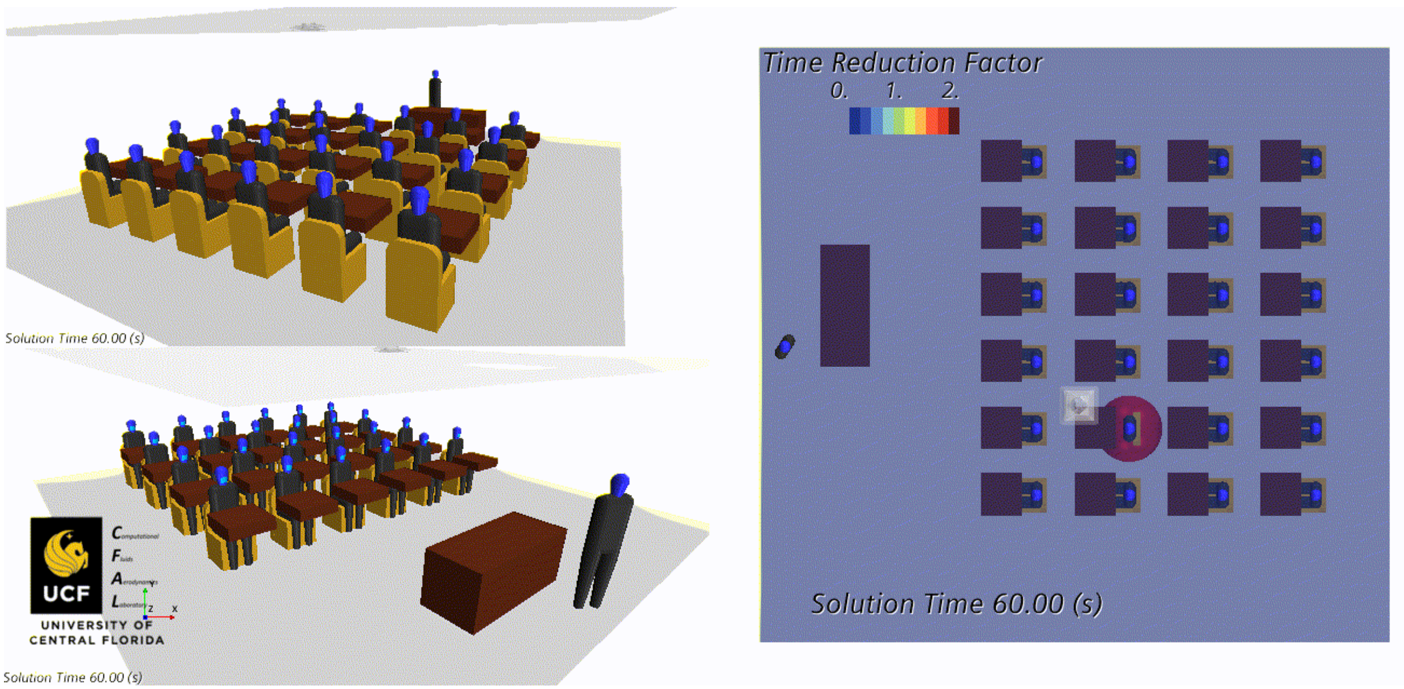


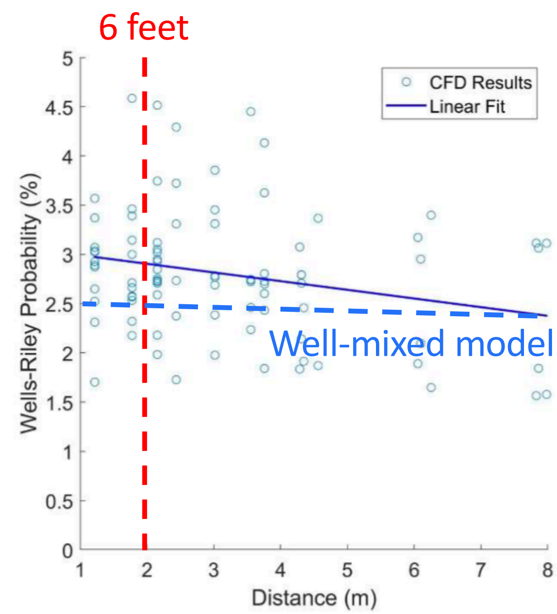
High risk: $\epsilon = 1\%$ Elderly: $s_r = 100\%$ Surgical masks: $p_m = 10\%$

Validation: 1/3 of global COVID-19 deaths in nursing homes

Beyond the Well Mixed Room

Natural Convection and Aerosol Mixing Indoors



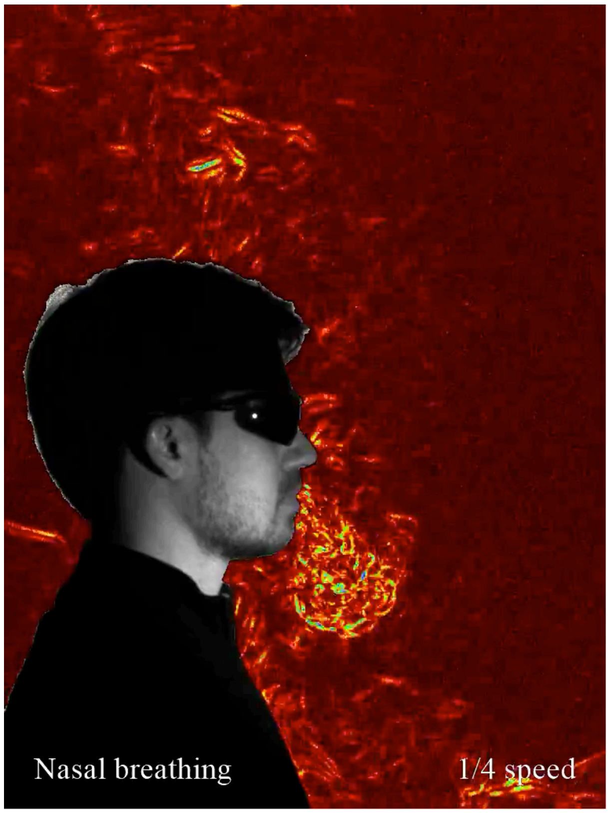


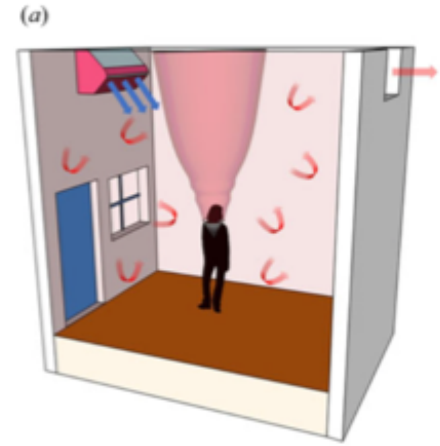
CFD simulation of a classroom (M. Kinzel, U. Central Florida)

Foster & M. Kinzel, Preprint (2021)

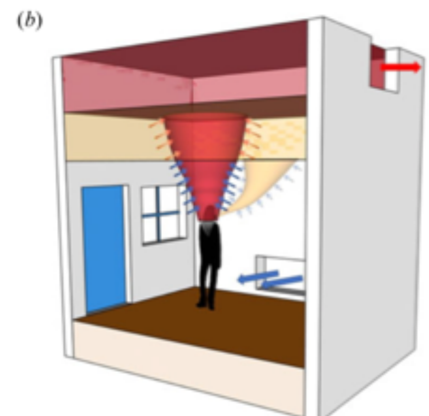
Respiration, Ventilation & Natural Convection

Bhagat, Wykes, Dalziel & Linden, J. Fluid Mech. (2020)





Mixing Ventilation



Displacement Ventilation

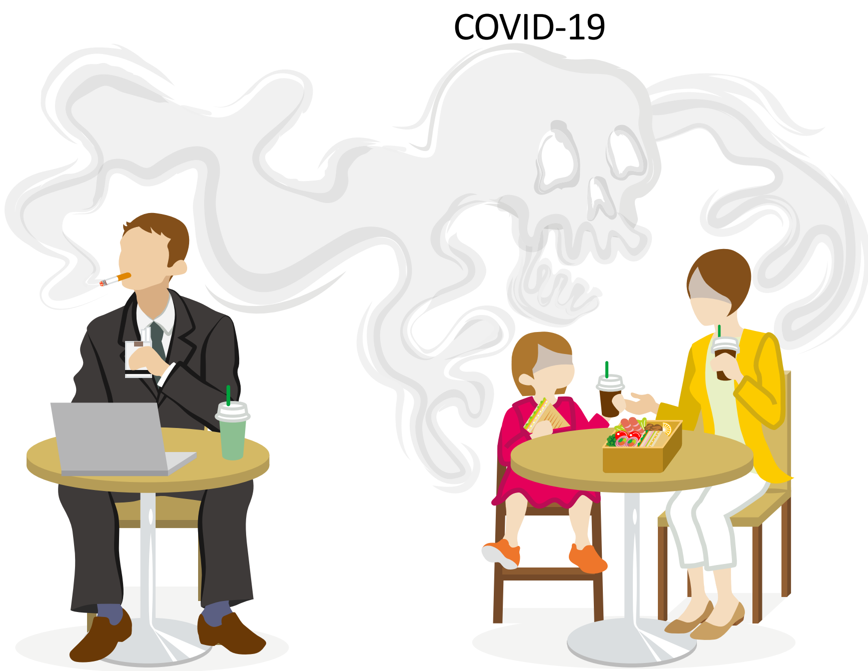
Two Types of Airborne Transmission

Short-range airborne transmission in turbulent plumes.

Long-range airborne transmission in a well-mixed room

https://tinyurl.com/FAQ-aerosols

We all understand airborne transmission.

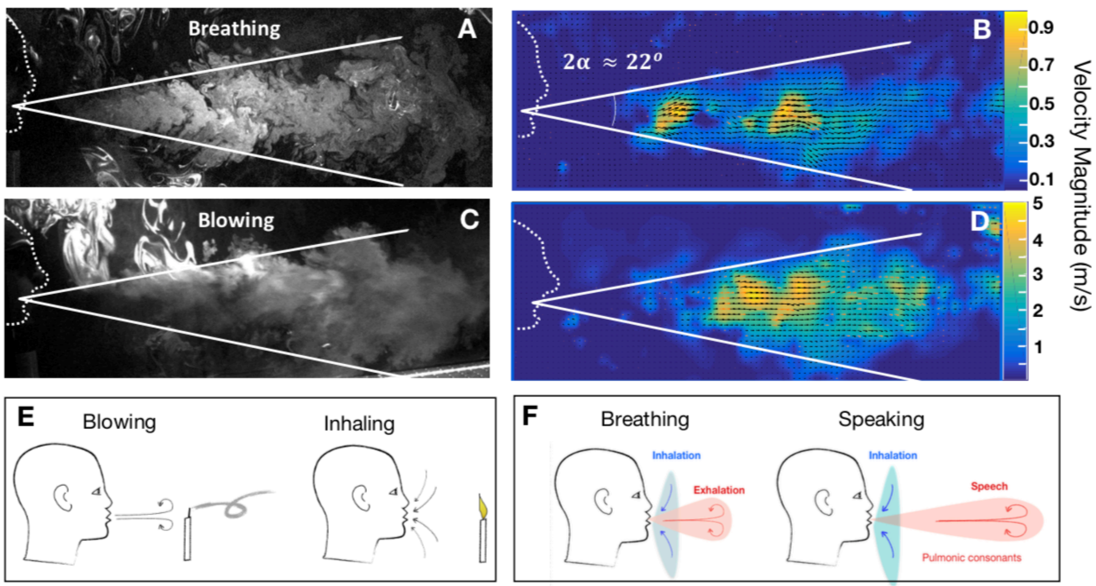


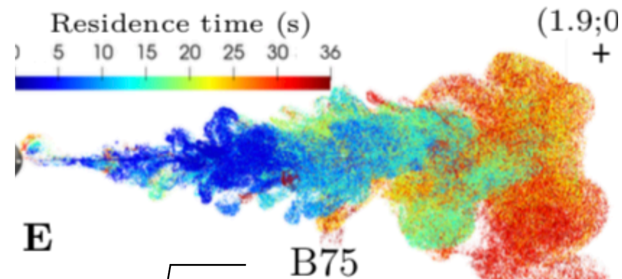
...and so can COVID-19 in a Korean apartment building!

Huang et al, Int. J. Infectious Diseases (2020)

Respiratory Turbulent Jets and Puff Trains

L. Bourouiba & J. W. M. Bush, *J. Fluid Mech.* (2014): Coughs and sneezes.
M. Abkarian, S. Mendez, N. Xue, F. Yang and H. A. Stone, *PNAS* (2020). Breathing and speaking.





$$\frac{C(x)}{C_0} \approx \frac{\sqrt{A_{mouth}}}{\alpha x}$$

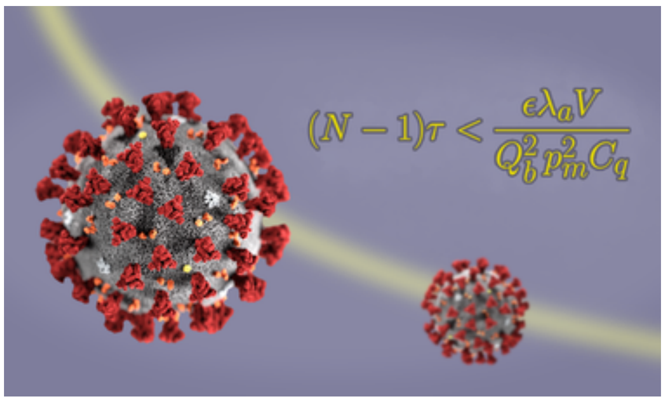
$$R_{in}(\tau)\left(1 + \frac{\lambda_c(\bar{r}) V \sqrt{A_{mouth}}\, p_{jet}}{(N-1) Q_b \alpha x}\right) < \epsilon$$

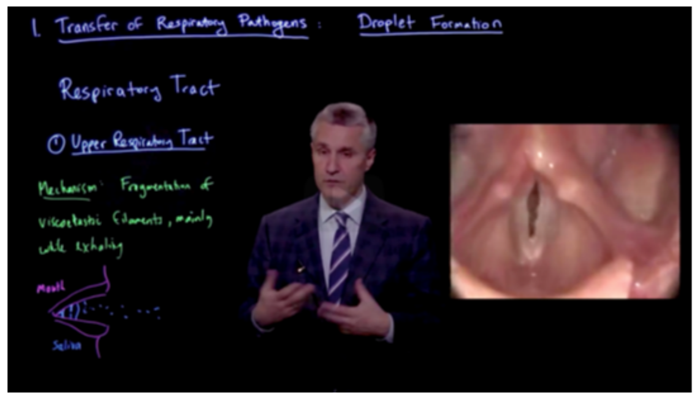
Non-universal excess risk from short-range transmission
→ Social distancing, if masks are not worn

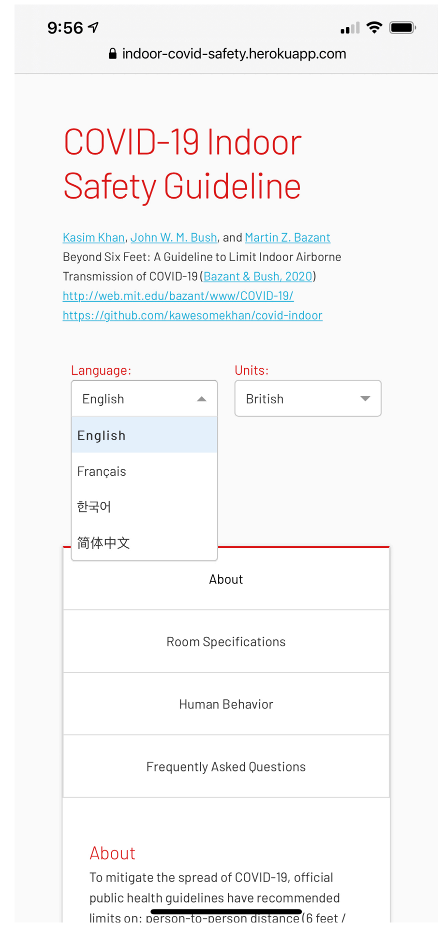
"Random packing" of room occupants: $p_{jet} \approx (\tan^{-1}\alpha)/\pi$ $x \approx \sqrt{A/N}$

COVID-19 Indoor Safety Guideline

Online app, spreadsheet: http://www.mit.edu/~bazant/COVID-19







- MOOC on edX: **10.S95x Physics of COVID-19 Transmission**
  (free, self-paced) **Enroll now!**
- Assignment: MOOC Ch. 4 Homework (turn in written solution)