

IAP 2021: THE NOTION OF ATOMICITY IN ALGEBRA AND COMBINATORICS

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PROBLEM SET

Problem 1. Consider the additive submonoid M of $(\mathbb{Q}_{\geq 0}, +)$ generated by the set $\{1/p : p \text{ is prime}\}$.

- (1) Find the set of atoms of M and prove that M is atomic.
- (2) Show that M satisfies the ascending chain condition on principal ideals.
- (3) Find an additive submonoid of $(\mathbb{Q}_{\geq 0}, +)$ that is atomic but does not satisfy the ascending chain condition on principal ideals.

Problem 2.

- (1) Show that the subring $\mathbb{Z}[x^2, x^3]$ of $\mathbb{Z}[x]$ is a bounded factorization domain that is not a half-factorial domain.
- (2) Show that the subring $\mathbb{R} + \mathbb{C}[x]$ of $\mathbb{C}[x]$ is a bounded factorization domain that is not a finite factorization domain.
- (3) Find a bounded factorization domain that is neither a half-factorial domain nor a finite factorization domain.

Problem 3.

- (1) Give an example of an atomic lattice that is not semimodular.
- (2) Give an example of a modular lattice that is not atomic.

Problem 4. Give an example of an infinite lattice that is a finite factorization lattice but not a unique factorization lattice.