Surface tension

John W. M. Bush

Department of Mathematics
MIT

The history of surface tension

First studies

- the dynamics of wine (Thomson 1855)
- modeling celestial bodies (Plateau 1873)

Geophysics

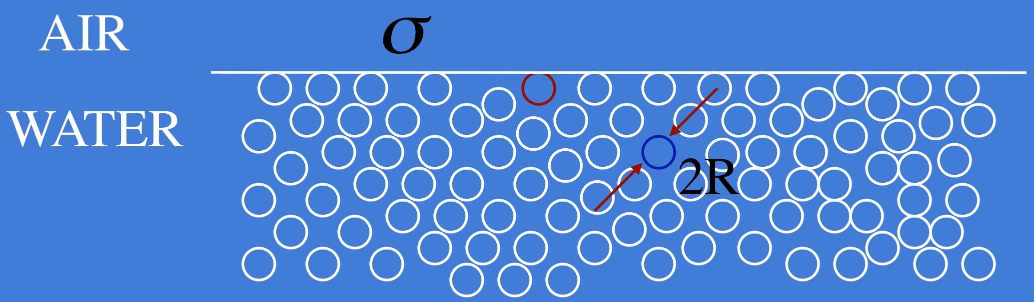
- the dynamics of raindrops
- mixing in the surf zone

Modern engineering applications

- inkjet printing, spray atomization
- manufacturing on the micro- and nanoscale
- biomimicry and water-repellant surfaces

Surface Tension: molecular origins

- each molecule in a fluid feels a cohesive force with surrounding molecules
- molecules at interface feel half this force; are in an energetically unfavorable state
- the creation of new surface is thus energetically costly



- cohesive energy per molecule of radius R in bulk is U, at surface is U/2
- surface tension is this loss of cohesive energy per unit area:

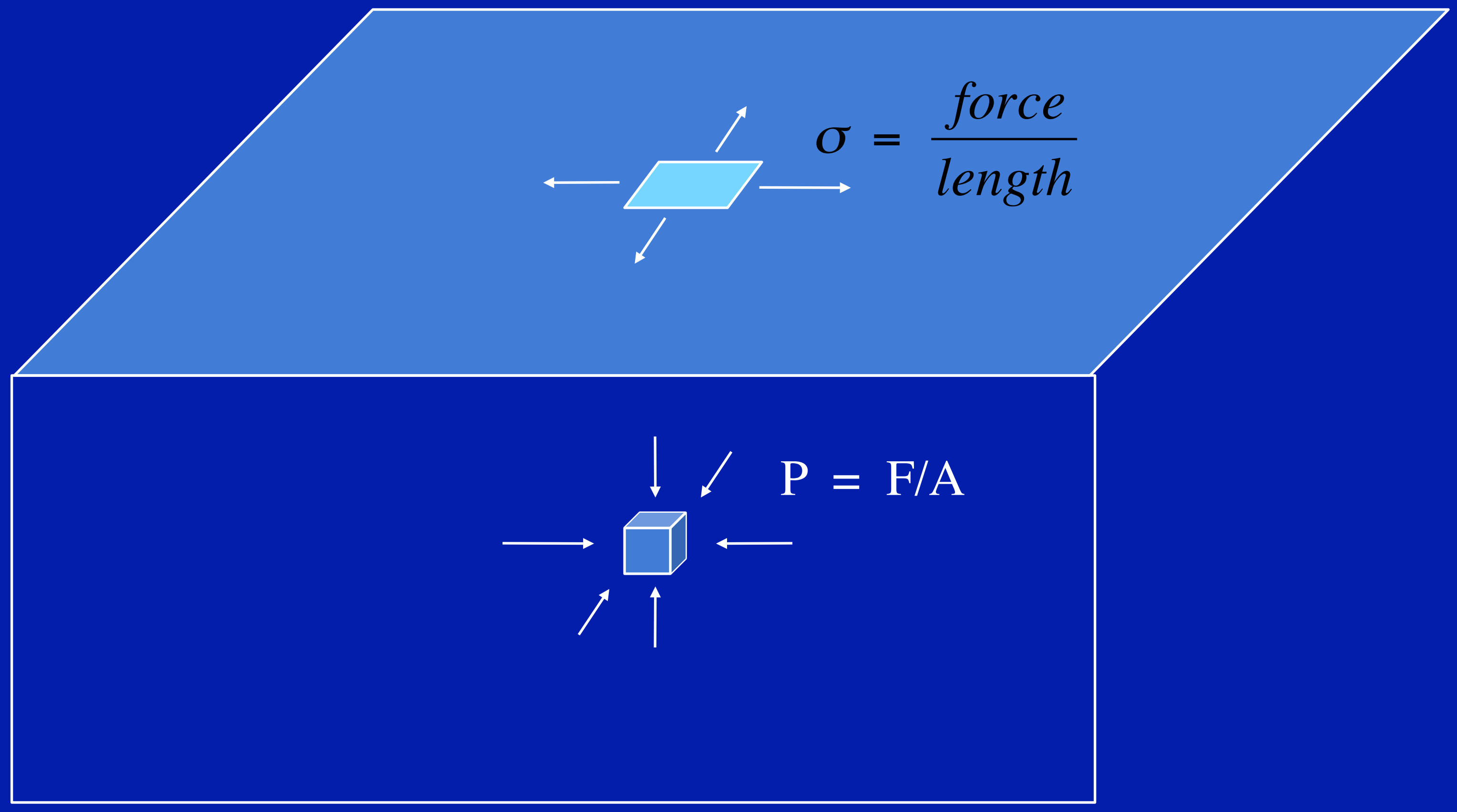
$$\boxed{\sigma \sim \frac{U}{R^2}}$$

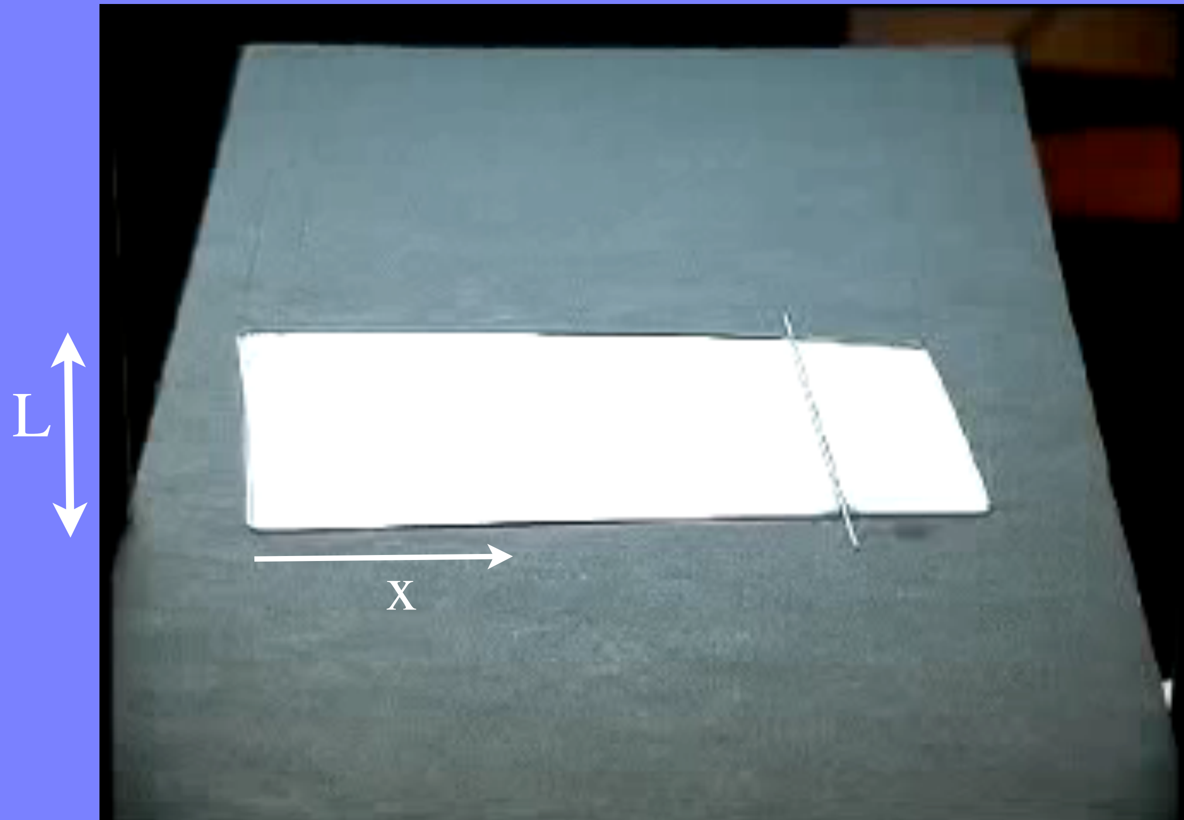
Units: $[\sigma] = \frac{\text{ENERGY}}{\text{AREA}} = \frac{\text{FORCE}}{\text{LENGTH}}$

- air-water $\sigma \sim 70$ dyne/cm; oils $\sigma \sim 20$ dyne/cm; liquid metals $\sigma \sim 500$ dyne/cm

Surface tension: analogous to a negative surface pressure

- gradients in surface tension necessarily drive surface motion





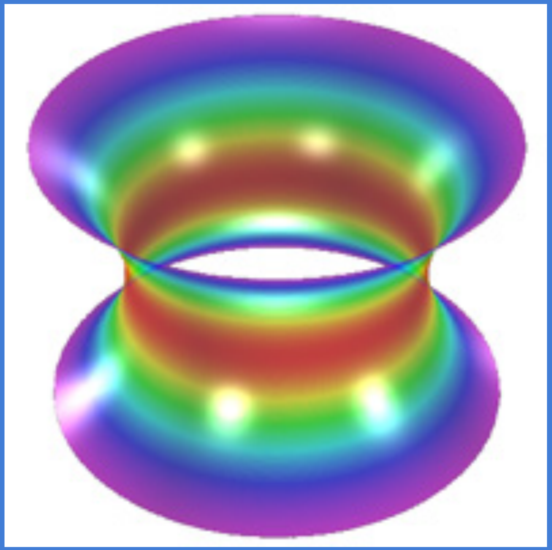
Surface tension: $$[\,\sigma\,] = \frac{FORCE}{LENGTH} = \frac{ENERGY}{AREA}$$

Surface energy: $$E_\sigma = \int_S \sigma \, dA = 2\,\sigma\,L\,x$$

Force acting on rod: $$F = \frac{dE_\sigma}{dx} = 2\,\sigma\,L$$

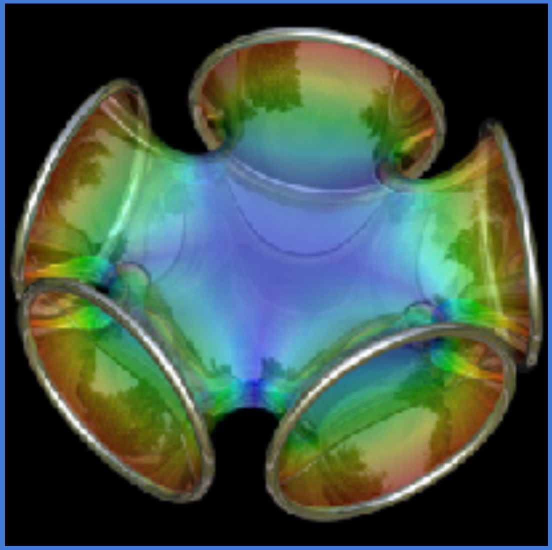
The creation of surface is energetically costly

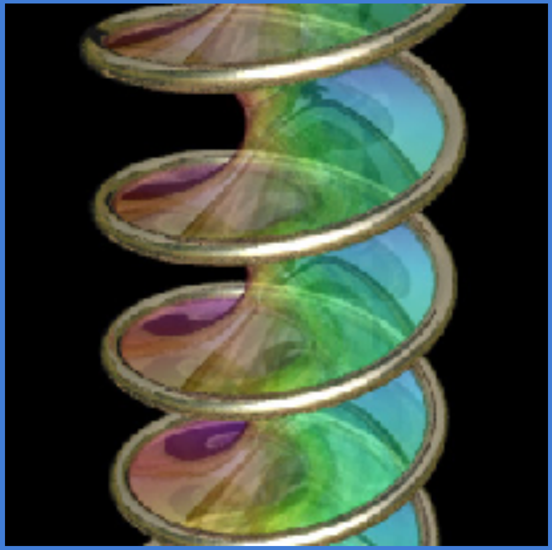
- quasi-static soap films (for which gravity, inertia are negligible) take the form of minimal surfaces

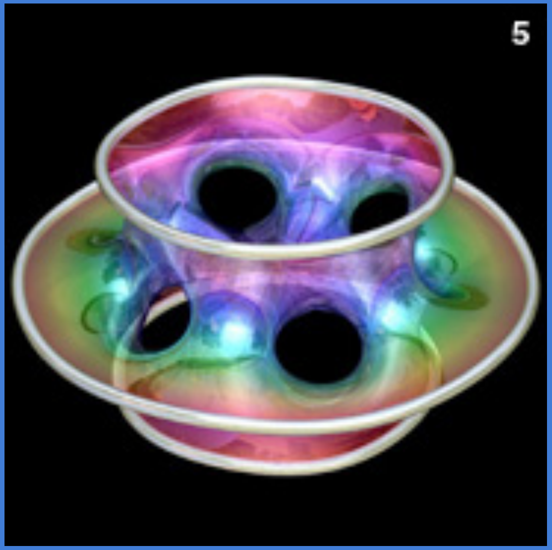


- hence their interest to mathematicians:

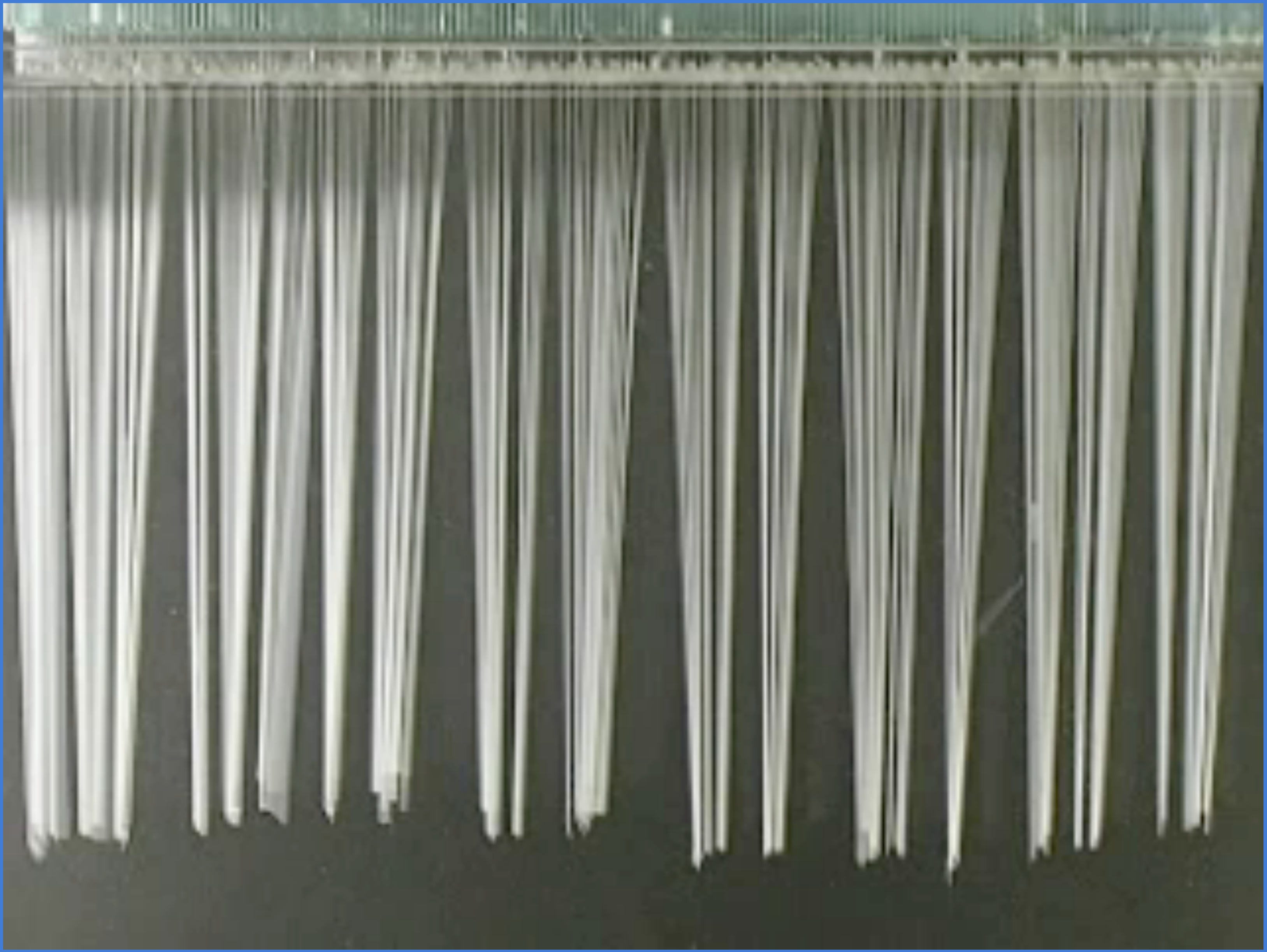
"Find the minimal surface bound by the multiply connected curve C, where C ...."







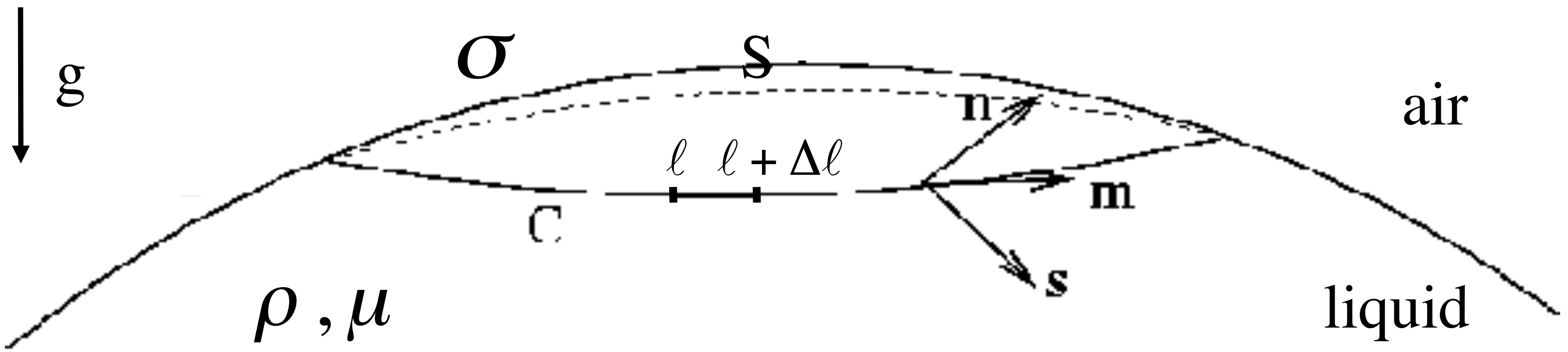
Minimal surfaces: surface energy minimized



The wet hair instability: threads clump to minimize surface energy

Surface tension: Geometry

Along a contour C bounding a surface S there is a tensile force per unit length σ acting in the $\mathbf{s}$ direction



Net force on S:

$$\boxed{\oint_C \sigma \, \mathbf{s} \; d\ell = \iint_S \sigma \, (\nabla \cdot \mathbf{n}) \, \mathbf{n} \; dS + \iint_S \nabla\sigma \; dS}$$

curvature pressure

Marangoni stress

1) normal curvature pressure $\sigma \nabla \cdot \mathbf{n}$ resists surface deformation

2) tangential Marangoni stresses may arise from $\nabla\sigma$

Governing Equations

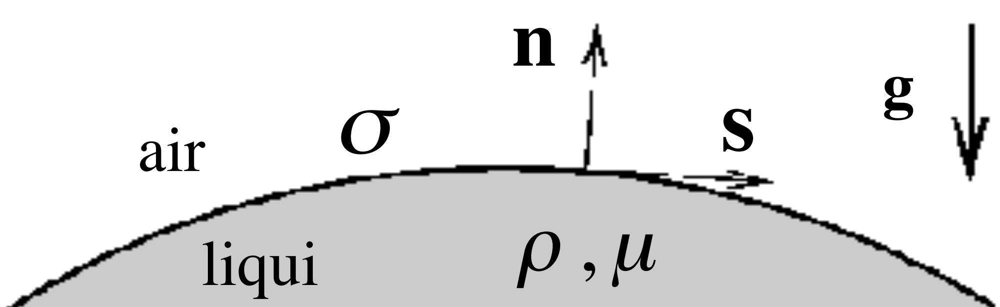
Navier-Stokes equations:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{u} \quad , \qquad \nabla\cdot\mathbf{u} = 0$$

Boundary Conditions

Normal stress: $$\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{n}\,\big| = \sigma\,\nabla\cdot\mathbf{n}$$

Tangential stress: $$\mathbf{n}\cdot\mathbf{T}\cdot\mathbf{s}\,\big| = \nabla_s\sigma$$



Stress tensor

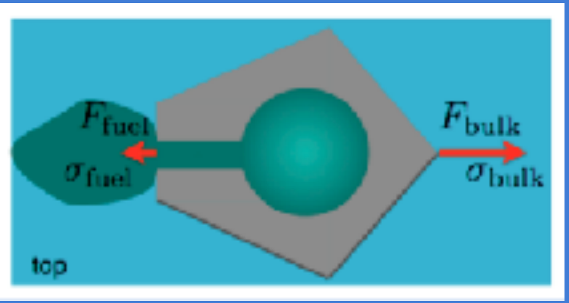
$$\mathbf{T} = -p\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$$

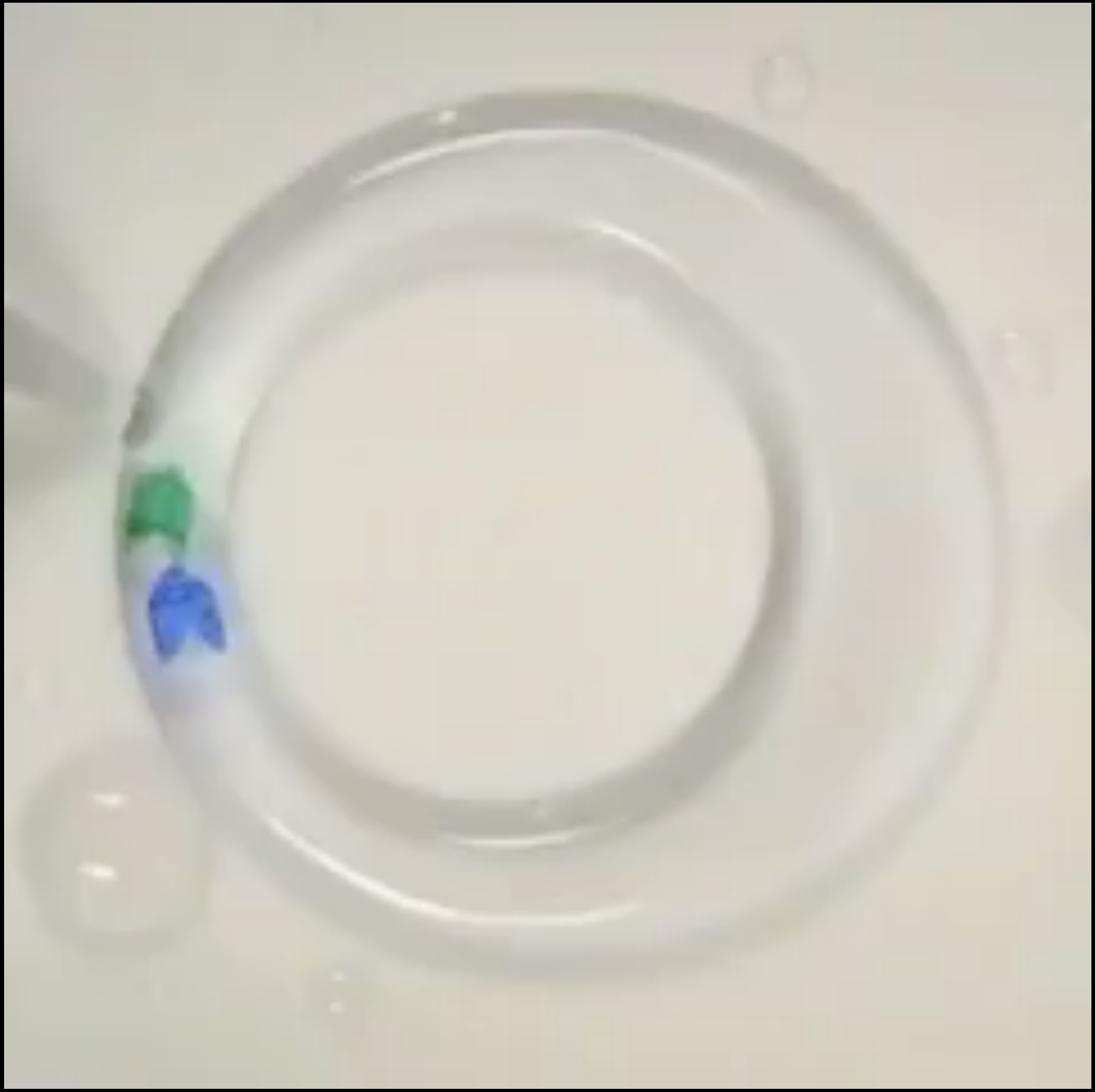
Curvature pressures, $\sigma \nabla \cdot \mathbf{n}$, make the surface behave as a trampoline.

The soap boat

- driven by a surface tension gradient

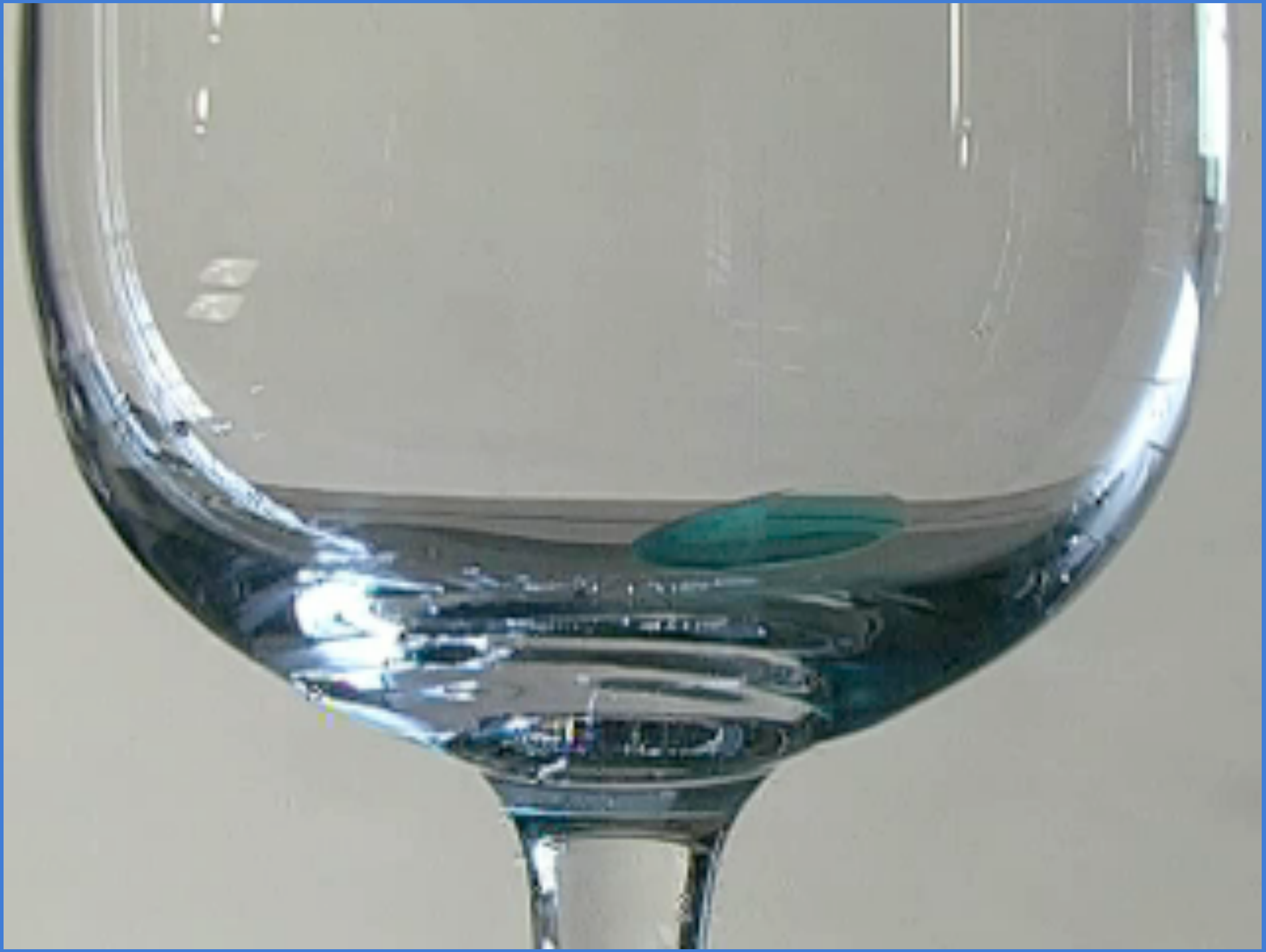
The cocktail boat:
fueled by alcohol



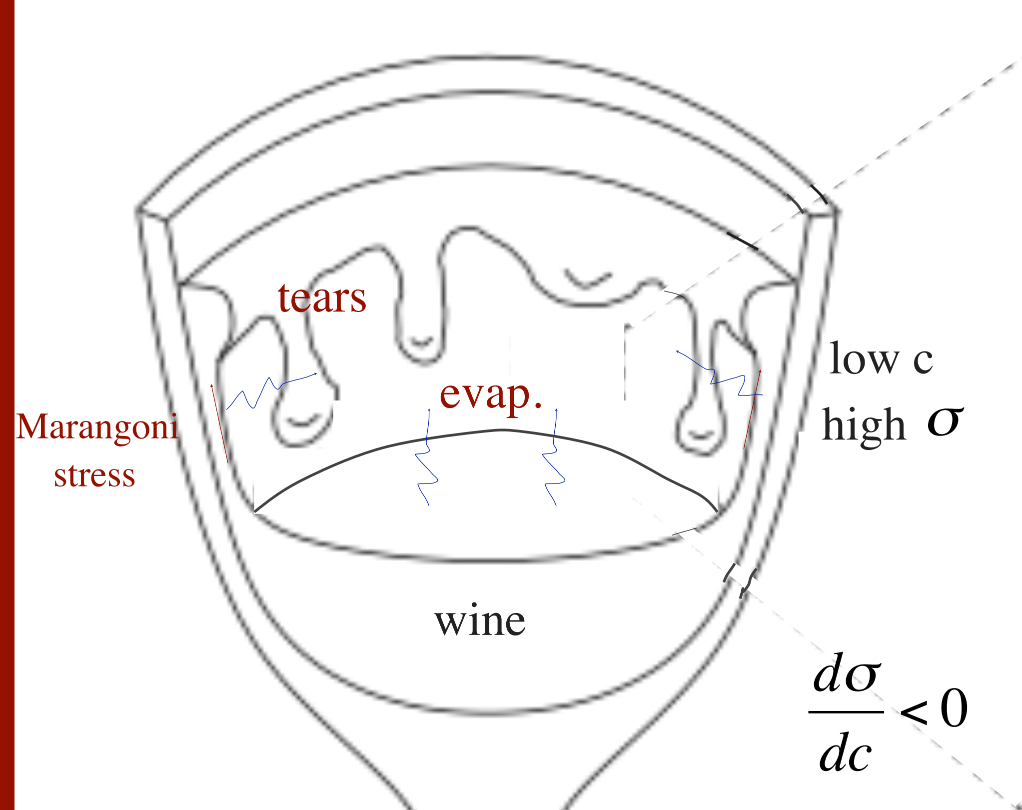


Nakata (2006)

The tears of wine



The first `Marangoni flow' studied scientifically (Thomson 1855).



The Tears of Wine

"Who hath sorrow? Who hath woe? They that tarry long at the wine.
Look not though upon the strong red wine that moveth itself aright.
At the last it biteth like a serpent and stingeth like an adder."

- Proverbs 23: 29-32 (c.a. 950 BC)

King Solomon, "the wisest man that ever lived".

When is surface tension important relative to gravity?

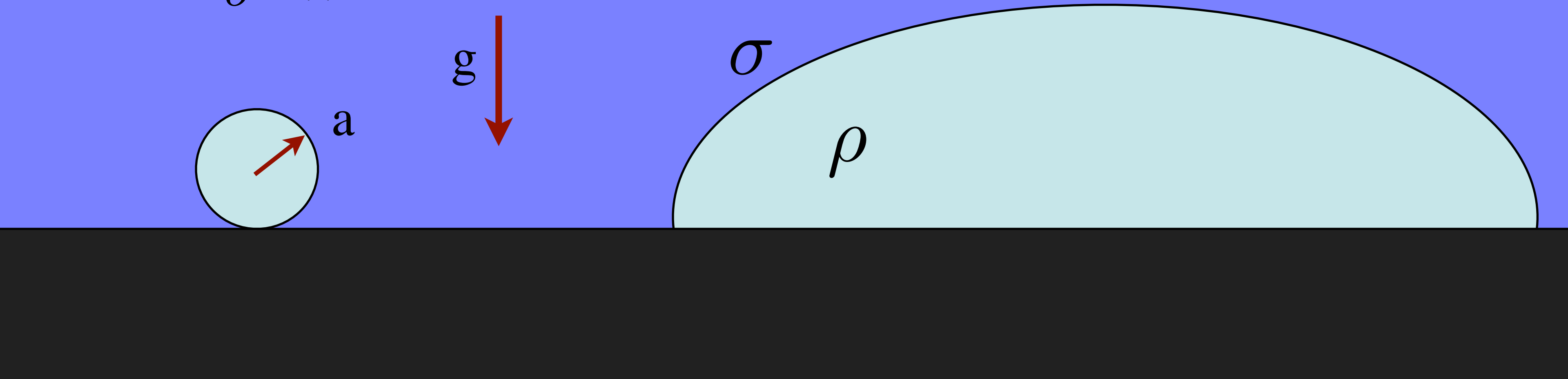
- when curvature pressures large relative to hydrostatic:

$$\text{Bond number:} \quad B_o = \frac{\rho g a^2}{\sigma} < 1$$

i.e. for drops small relative to the capillary length:

$$\boxed{a < l_c = \left(\frac{\sigma}{\rho g}\right)^{1/2}} \sim 2 \text{ mm for air-water}$$

$B_o \ll 1$ $\qquad$ $B_o > 1$



Falling drops

Force balance:

$$\rho_a U^2 a^2 \sim Mg = \frac{4}{3}\pi a^3 \rho g$$

Fall speed: $U \sim \left(\frac{\rho}{\rho_a} g a\right)^{1/2}$

Drop integrity requires:

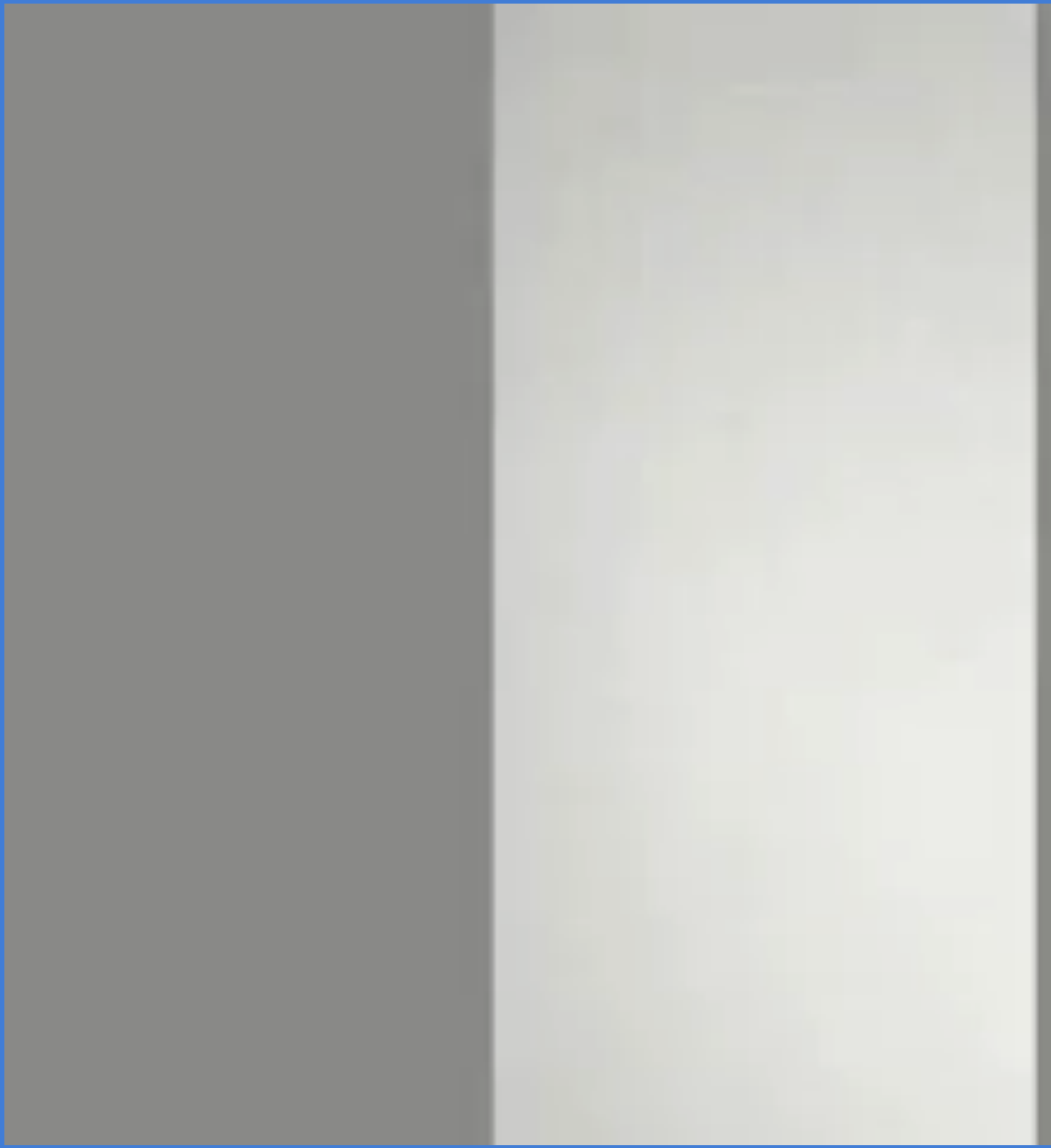
$$\rho_a U^2 / a = \rho g a < \sigma / a$$

Small drops

If a drop is small relative to the capillary length

$$a < \ell_c = \sqrt{\sigma / \rho g} \approx 2\text{mm} ,$$

σ maintains it against the destabilizing influence of aerodynamic stresses.



Big drops

Drops larger than the capillary length

$$a > \ell_c \approx 2\text{mm}$$

break up under the influence of aerodynamic stresses.

The break-up yields drops with size of order:

$$\ell_c \approx 2\text{mm}$$

Fluid statics and the curvature force

Recall

Normal stress condition: $\Delta\, \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{n} = \sigma \, (\nabla \cdot \mathbf{n})$

Tangential stress condition: $\Delta\, \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{t} = \nabla \sigma$

where $\mathbf{T} = -p\mathbf{I} + \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$

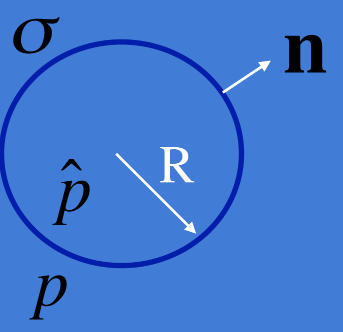
Fluid Statics $\quad \mathbf{T} = -p\mathbf{I} \;, \quad \hat{\mathbf{T}} = -\hat{p}\,\mathbf{I}$

Normal stress balance: $\quad \hat{p} - p = \sigma \nabla \cdot \mathbf{n}$

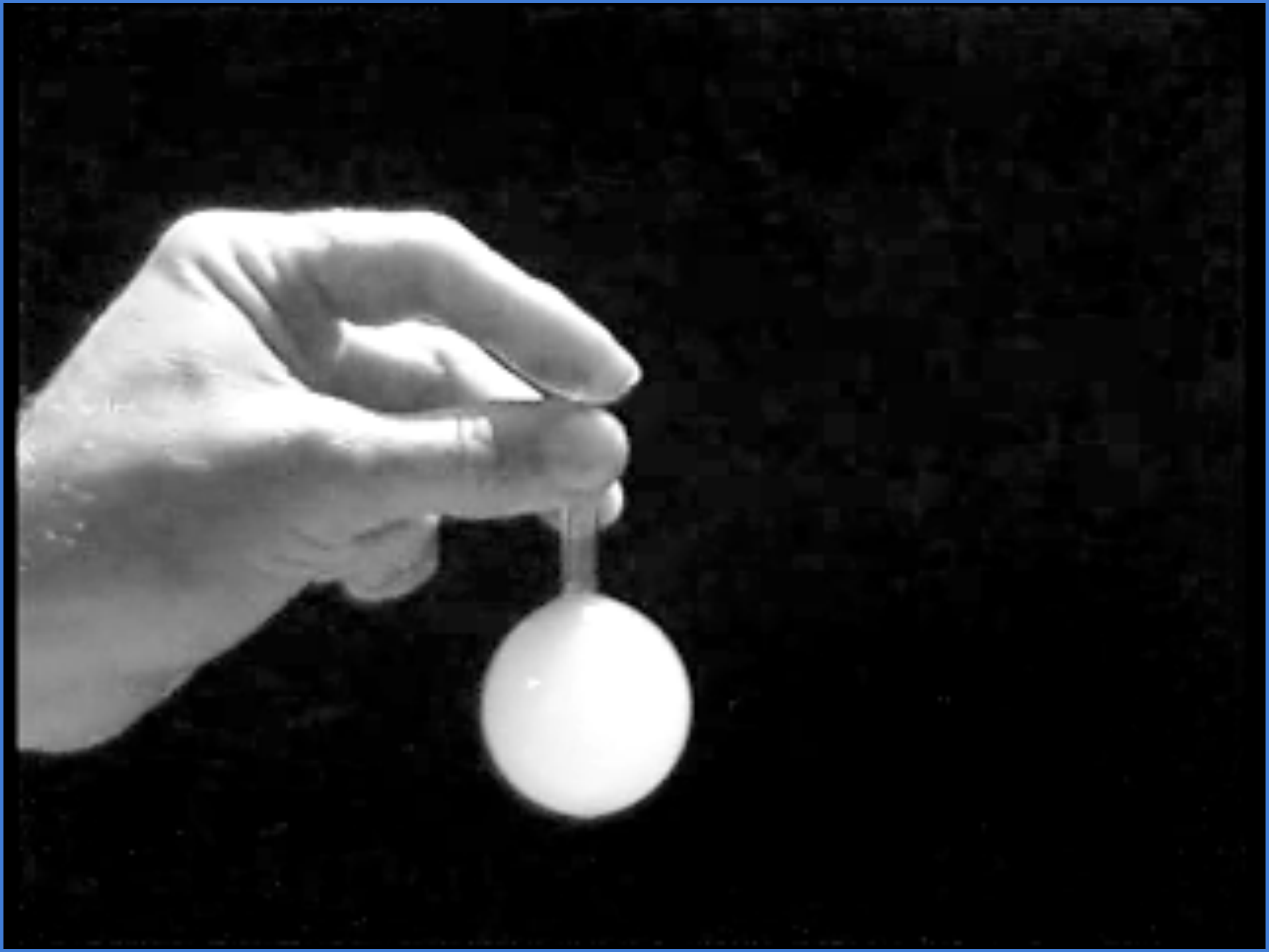
Tangential stress balance: $\quad 0 = \nabla \sigma$

Stationary bubble: what is the pressure drop across a bubble surface?

$$\hat{p} - p = \sigma \nabla \cdot \mathbf{n} = \frac{2\sigma}{R}$$



- smaller bubbles burst more loudly than large ones
- champagne is louder than beer

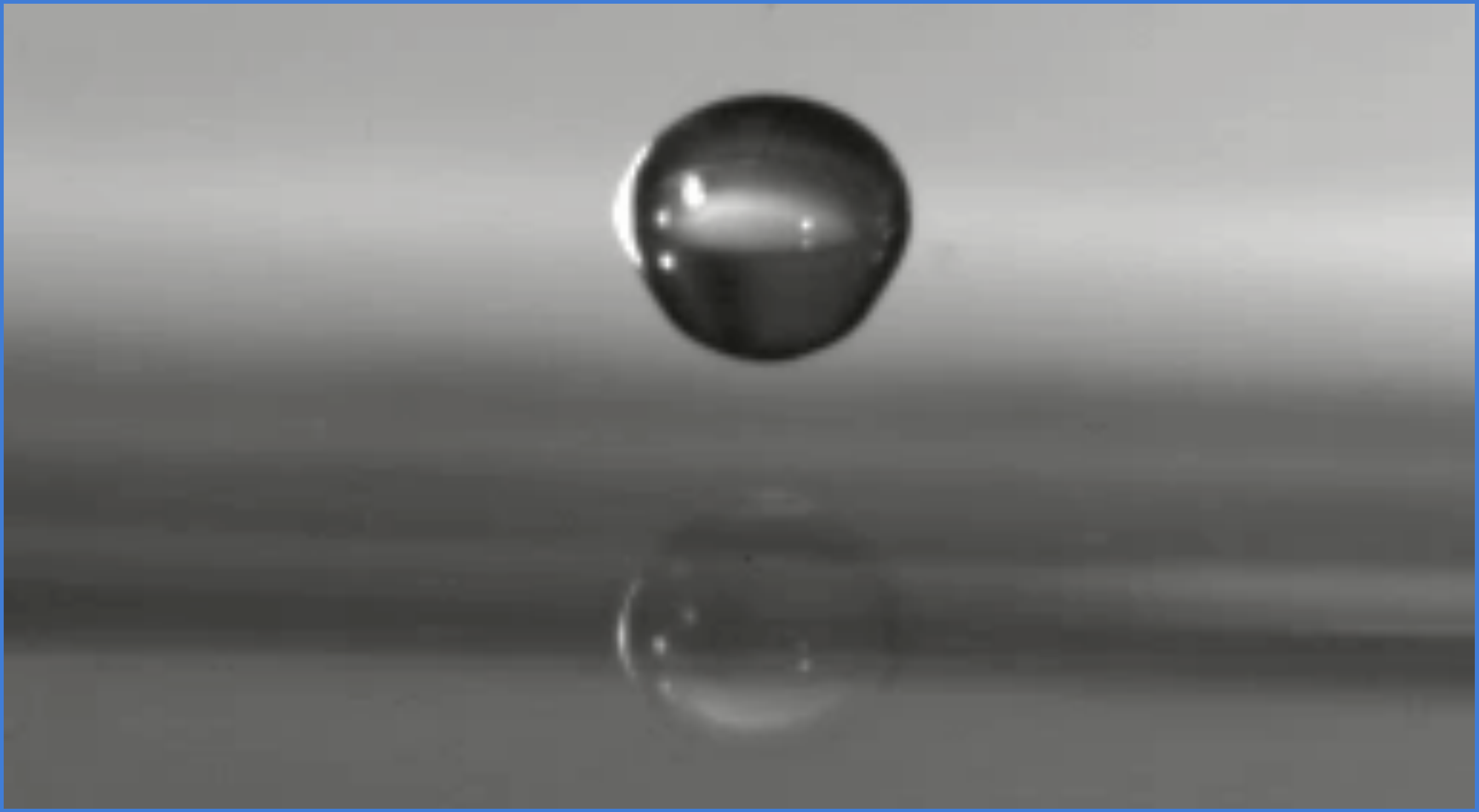


Capillary pressure

Which way does the air go?

Who cares about surface tension?

Raindrops strike a puddle

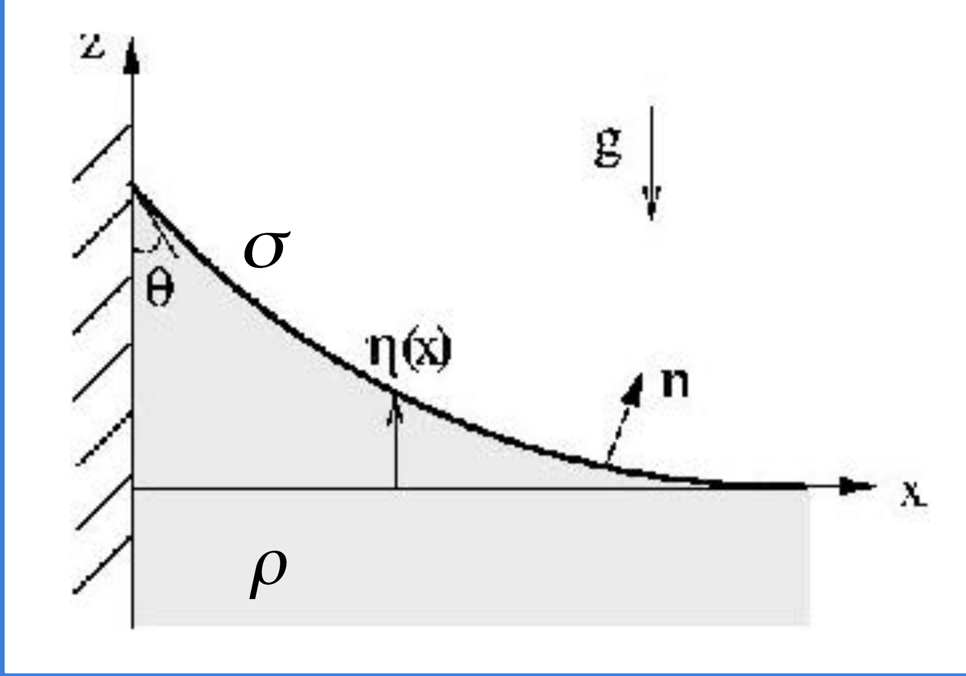


The coalescence cascade.

Static 2D meniscus

Young-Laplace equation

$$\boxed{\rho g \eta = \sigma \nabla \cdot \mathbf{n}}$$



Define: $\mathrm{f(x,z)} = \mathrm{z} - \eta(\mathrm{x})$

so that $\mathbf{n} = \dfrac{\nabla \mathrm{f}}{|\nabla \mathrm{f}|} = \dfrac{\mathbf{z} - \eta_{\mathrm{x}}(\mathrm{x})\mathbf{x}}{[1+\eta_{\mathrm{x}}^2(\mathrm{x})]^{1/2}}$

and $\nabla \cdot \mathbf{n} = \dfrac{\eta_{\mathbf{xx}}}{(1+\eta_{\mathbf{x}}^{\mathbf{2}})^{3/2}} \approx \eta_{\mathrm{xx}}$ provided $|\eta_{\mathrm{x}}| << 1$

Linearized: $\rho g \eta(\mathrm{x}) = \sigma \eta_{\mathbf{xx}}$ BCs: $\eta_{\mathrm{x}}(0) = -\cot\theta$, $\eta(\infty) = 0$

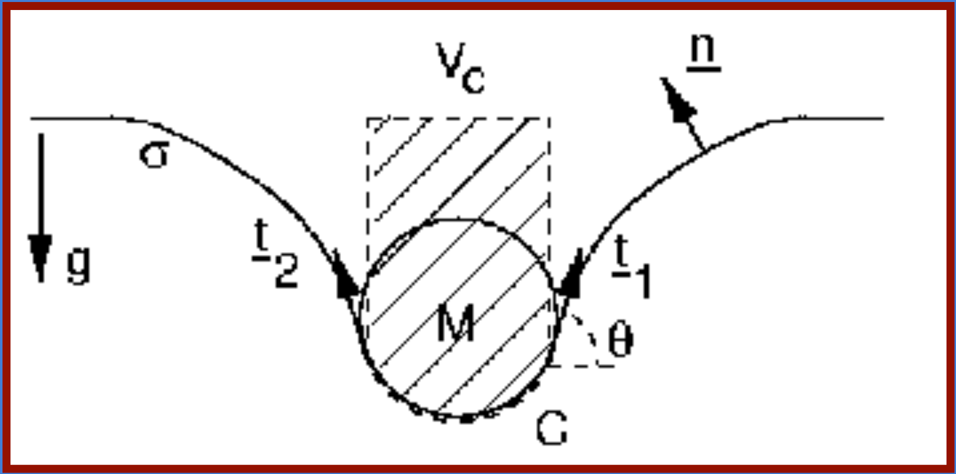
$$\longrightarrow \boxed{\eta(x) = \ell_c \cot\theta \exp(-x/\ell_c)}$$

Meniscus decays exponentially over the capillary length $\ell_c = \left(\dfrac{\sigma}{\rho g}\right)^{1/2}$

Heavy things sink, light things float.

Not exactly…..

Statics of floating bodies



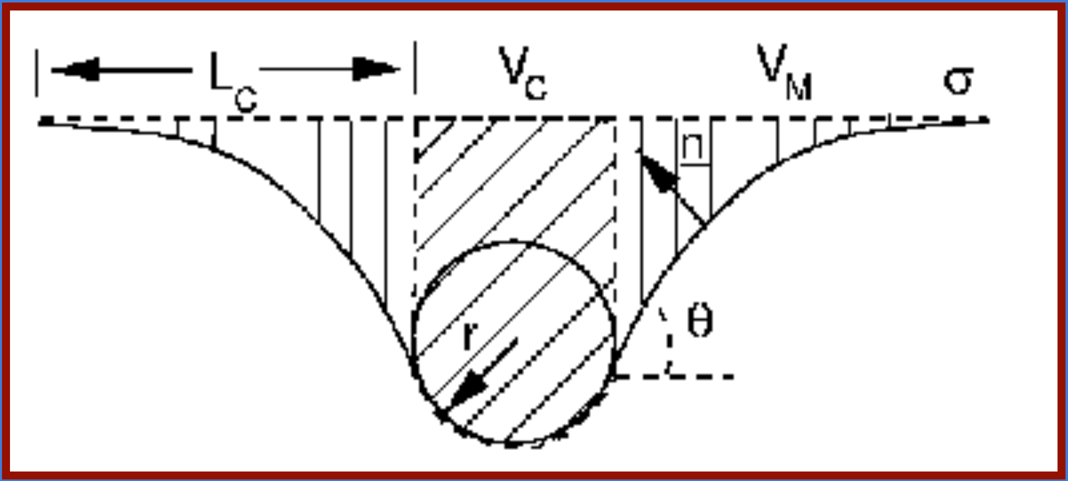
Force balance on body:

$$Mg = \int_C -p\, \hat{\mathbf{n}} \cdot \hat{\mathbf{z}}\, d\ell = F_b + F_c$$

Buoyancy: $$F_b = \int_C \rho\, g\, z\, (\hat{\mathbf{n}} \cdot \hat{\mathbf{z}})\, d\ell = \rho\, g\, V_c$$

Curvature: $$F_c = \int_c \sigma\, \nabla \cdot \hat{\mathbf{n}}\, (\hat{\mathbf{n}} \cdot \hat{\mathbf{z}})\, d\ell = \sigma\left(\hat{\mathbf{t}}_1 + \hat{\mathbf{t}}_2\right) \cdot \hat{\mathbf{z}} = 2\sigma \sin\theta$$

via Frenet-Serret equation: $$(\nabla \cdot \hat{\mathbf{n}})\, \hat{\mathbf{n}} = \frac{d\hat{\mathbf{t}}}{d\ell}$$

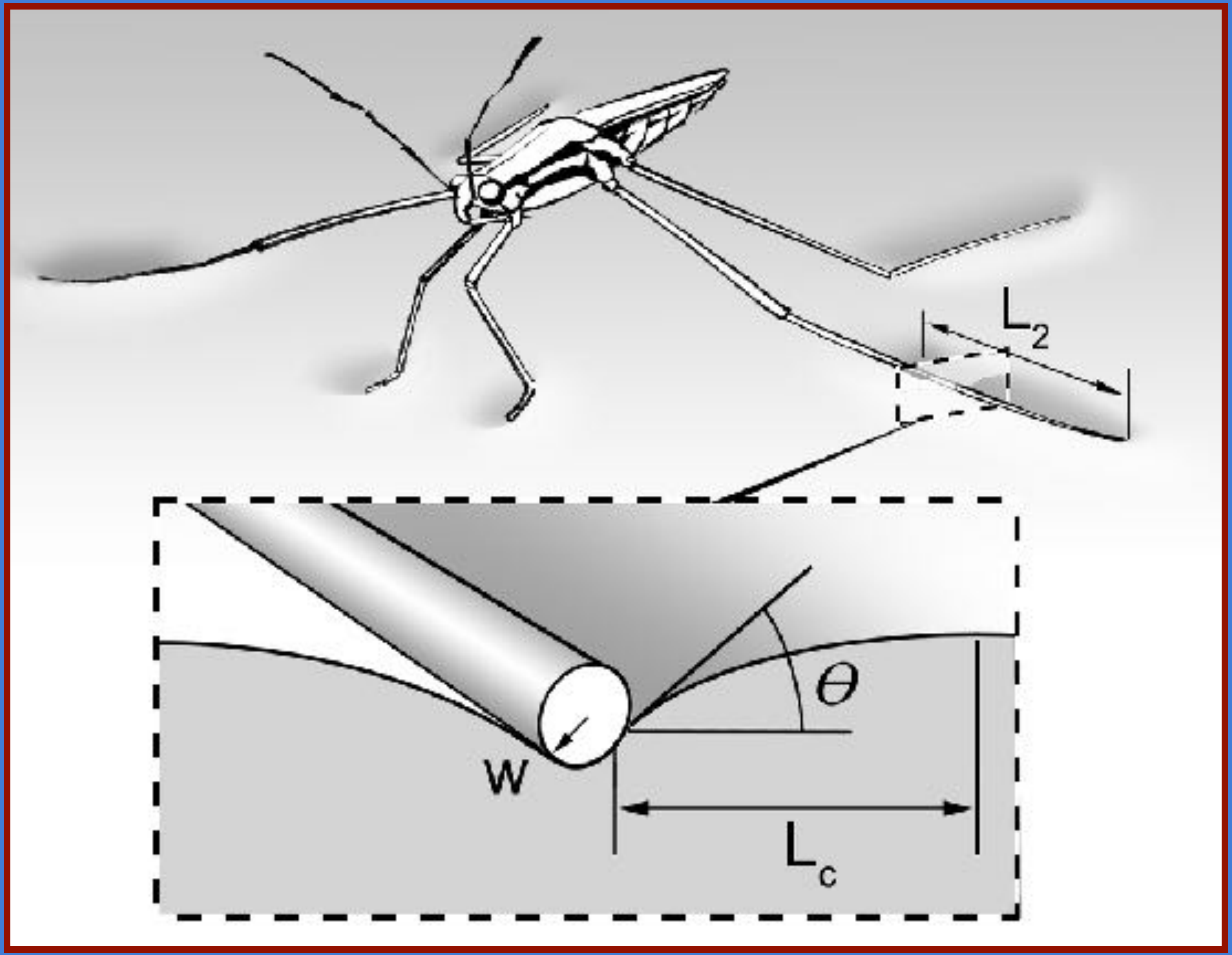


$$\Rightarrow \; F_b = \rho\, g\, V_c =$$ wt. of fluid displaced above body

$$\Rightarrow F_c = 2\sigma \sin\theta = \rho g V_M =$$ wt. of fluid above meniscus

$$\Rightarrow \boxed{\frac{F_b}{F_c} = \frac{V_c}{V_M} \approx \frac{r}{L_c}}$$ where $$L_c = \left(\frac{\sigma}{\rho g}\right)^{1/2} \approx 0.3\,\text{cm}$$

$\Rightarrow$ small objects (eg. insects) can be supported by σ



Static weight support requires: $M\, g < 2\,\sigma\, P \cos\theta$

where P is total contact length

Surface Tension in Biology

In all things of Nature, there is something of the marvelous.

- Aristotle

Motivation: who cares about surface tension?

As we have seen, surface tension dominates gravity on a scale less than the capillary length, ~2 mm.

Biology

- all small creatures live in a world dominated by surface tension
- surface tension important for insects for many basic functions
- weight support and propulsion at the water surface
- adhesion and deadhesion via surface tension
- the pistol shrimp: hunting with bubbles (VIDEO)
- the archer fish: hunting with drops
- underwater breathing and diving via surface tension
- natural strategies for water-repellency in plants and animals
- the hydraulics of trees

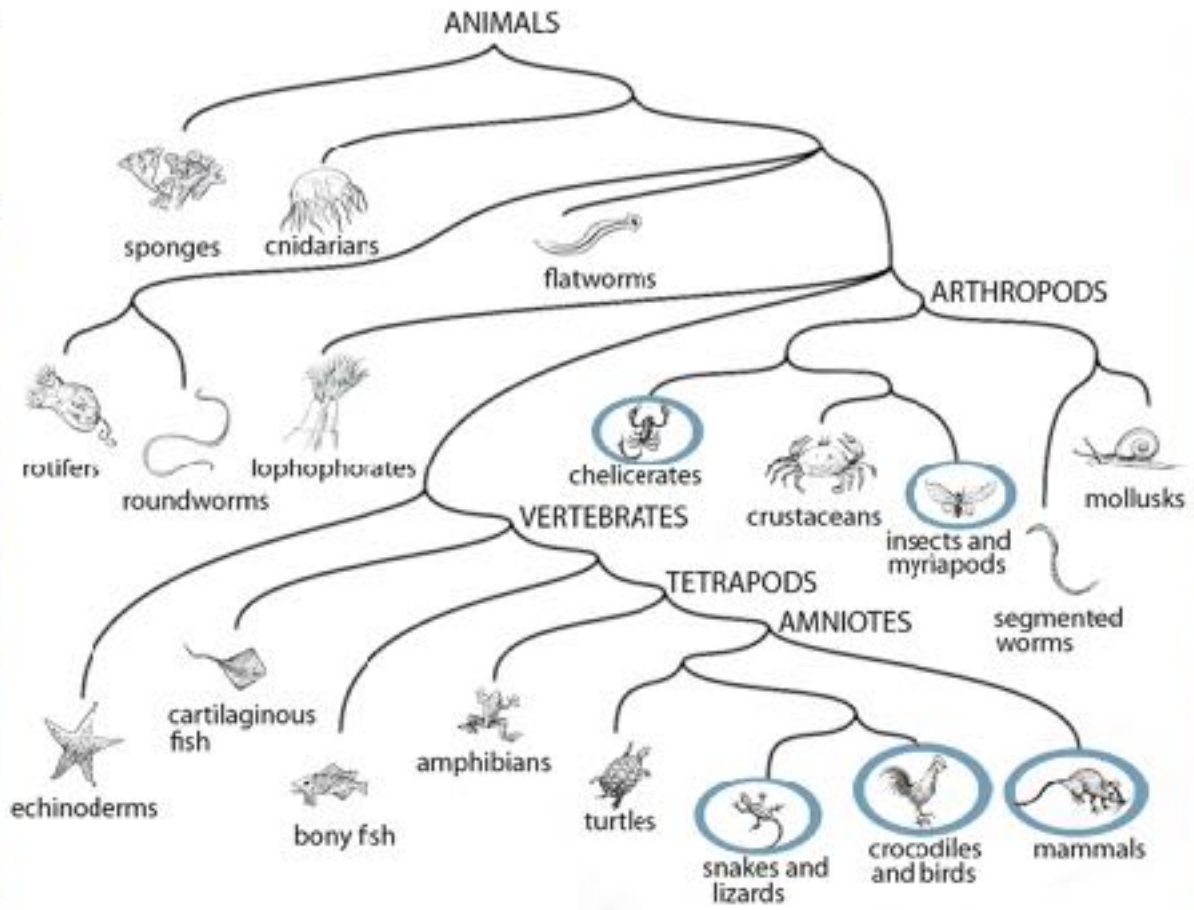
Hunting with bubbles

The Pistol Shrimp

Walking on water

Water-walkers in the tree of life (over 1200 species)



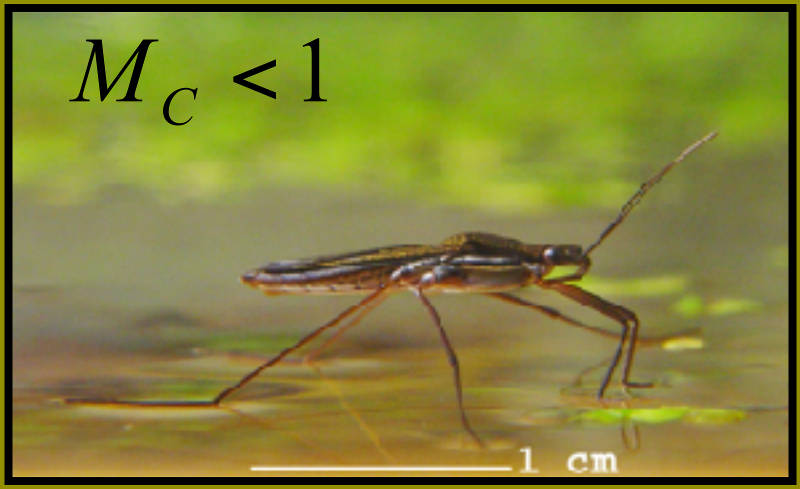


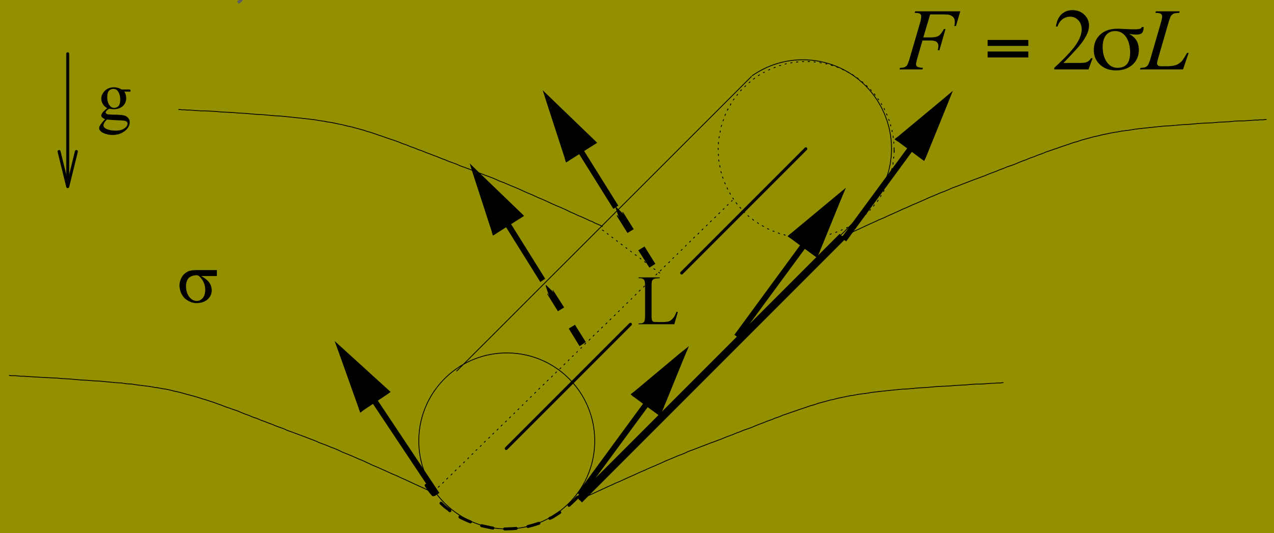

Motivation: foraging on water surface, avoidance of predators

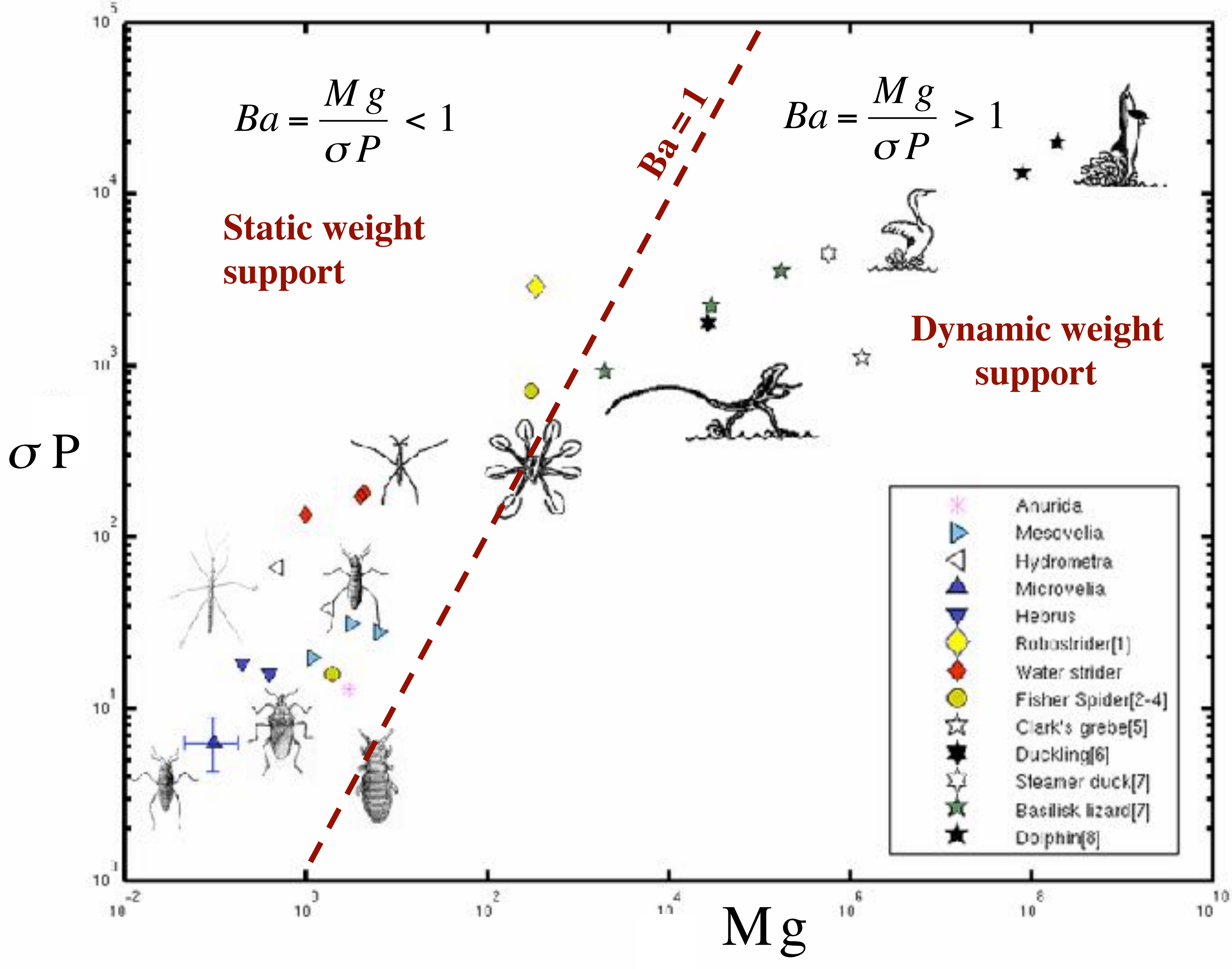
Water-walking

$$M_C = \frac{Mg}{2\sigma P} = \frac{\text{weight}}{\text{surface tension force}}$$

McMahon, 1996







$Ba = \frac{Mg}{\sigma P} < 1$
Ba = 1
$Ba = \frac{Mg}{\sigma P} > 1$
Static weight support
Dynamic weight support
σ P
Mg
Anurida
Mesovelia
Hydrometra
Microvelia
Hebrus
Robostrider[1]
Water strider
Fisher Spider[2-4]
Clark's grebe[5]
Duckling[6]
Steamer duck[7]
Basilisk lizard[7]
Dolphin[8]

Biological classification

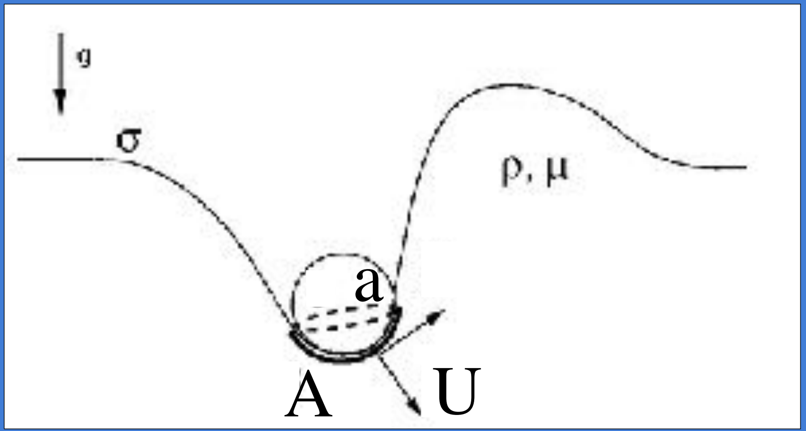
- made along evolutionary grounds

Dynamic classification

- group according to propulsion mechanism

- evaluate relative magnitudes of hydrodynamic forces

Dynamic classification of water-walkers



$$F_H \sim \rho g V_s + \rho U^2 A + \rho V \frac{dU}{dt} + \rho \nu U a + \sigma\left(\underline{\nabla} \cdot \underline{n}\right) A - \underline{\nabla}\sigma A$$

buoyancy — form drag — acceleration reaction — viscous drag — curvature — Marangoni

Mathematician: which terms can I discard to get a tractable equation?

Physicist: which forces are used by which creatures?

→ every force is used by some creature

	$\rho g z\ A$	$\rho V\, dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n}\ A$	$\underline{\nabla} \sigma\ A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

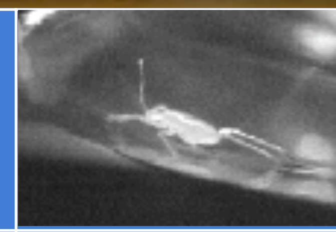
Clark's Grebe: clip courtesy of "Winged Migration"

Common Skittering Frog; Walking on water

BBC's Natural World, April 2013

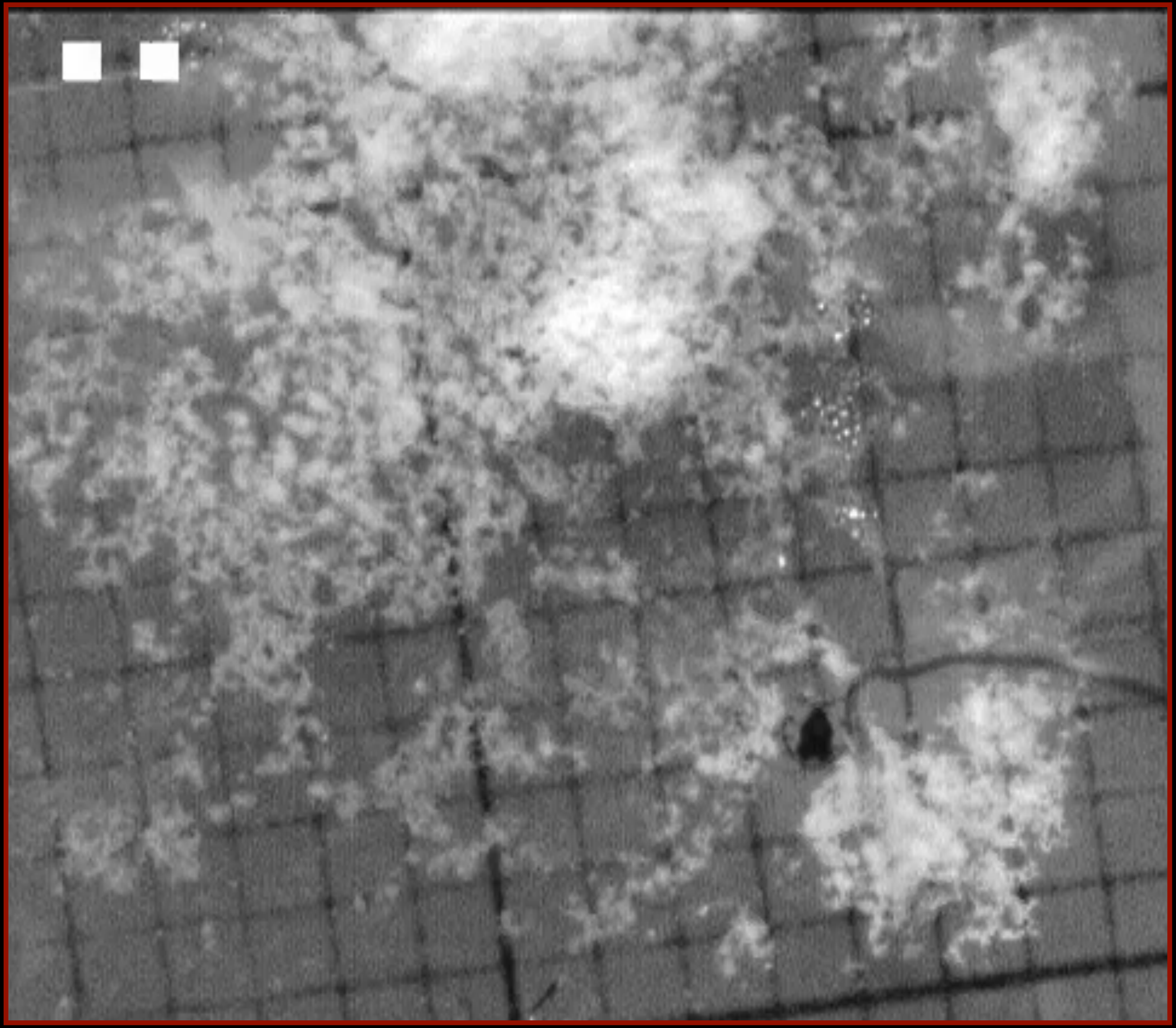
Courtesy of National Geographic


Video by Tonia Hsieh, Lauder
Laboratory Harvard University

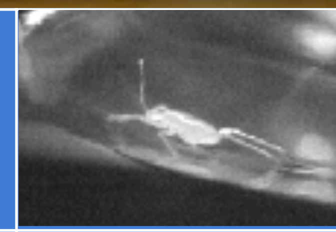
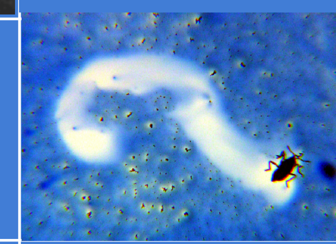
	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

quasi – static propulsion

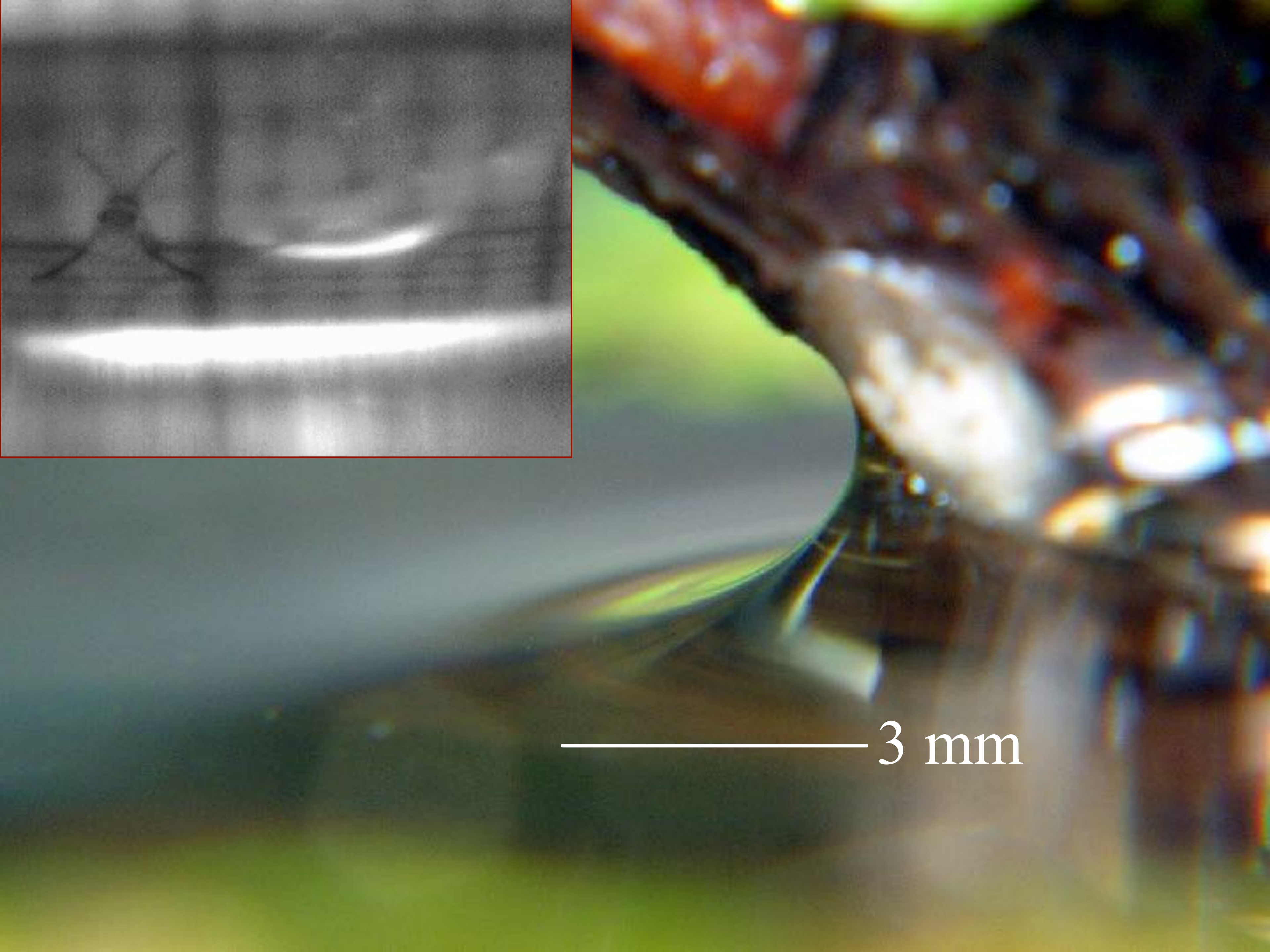
Tangential stress, $\boxed{\nabla\sigma}$ may drive lateral motion.



Marangoni propulsion: insect uses lipid as fuel.

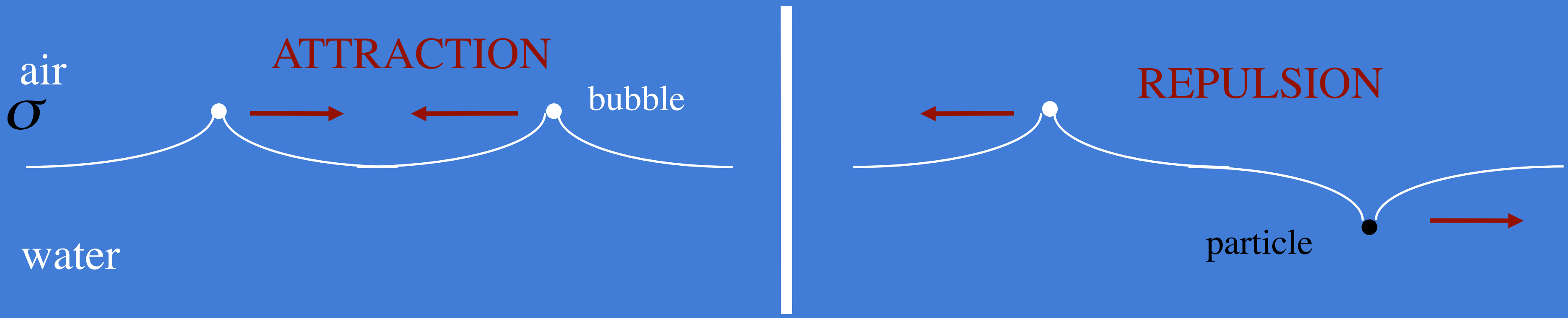
	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

quasi – static propulsion


3 mm

Capillary forces: The Cheerios effect

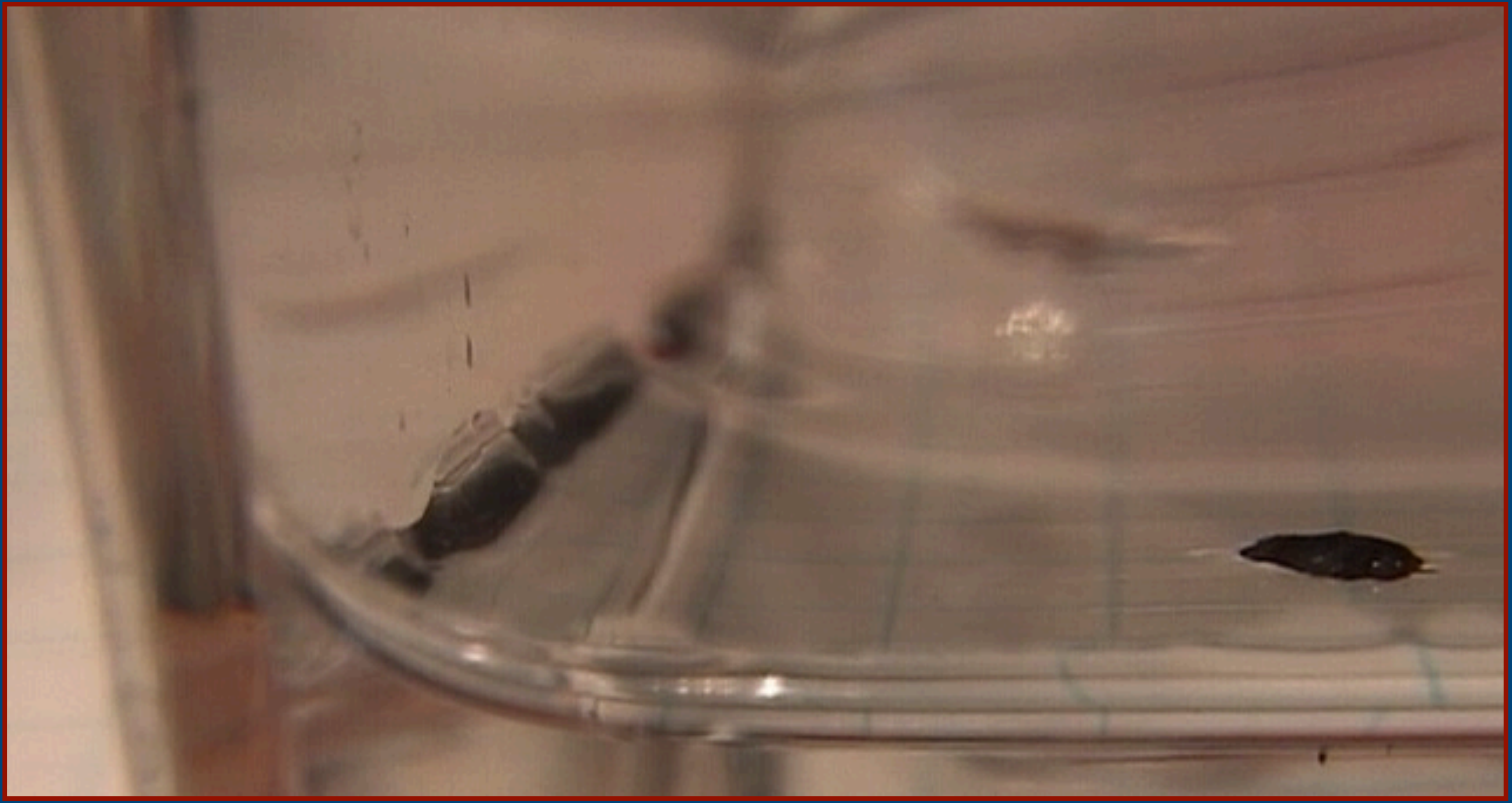
- exist between objects floating at a free surface
- attractive/repulsive for meniscii of the same/opposite sense



- explains the formation of bubble rafts in champagne
- explains the attraction of Cheerios in a bowl of milk
- used by small insects to move themselves along the free surface

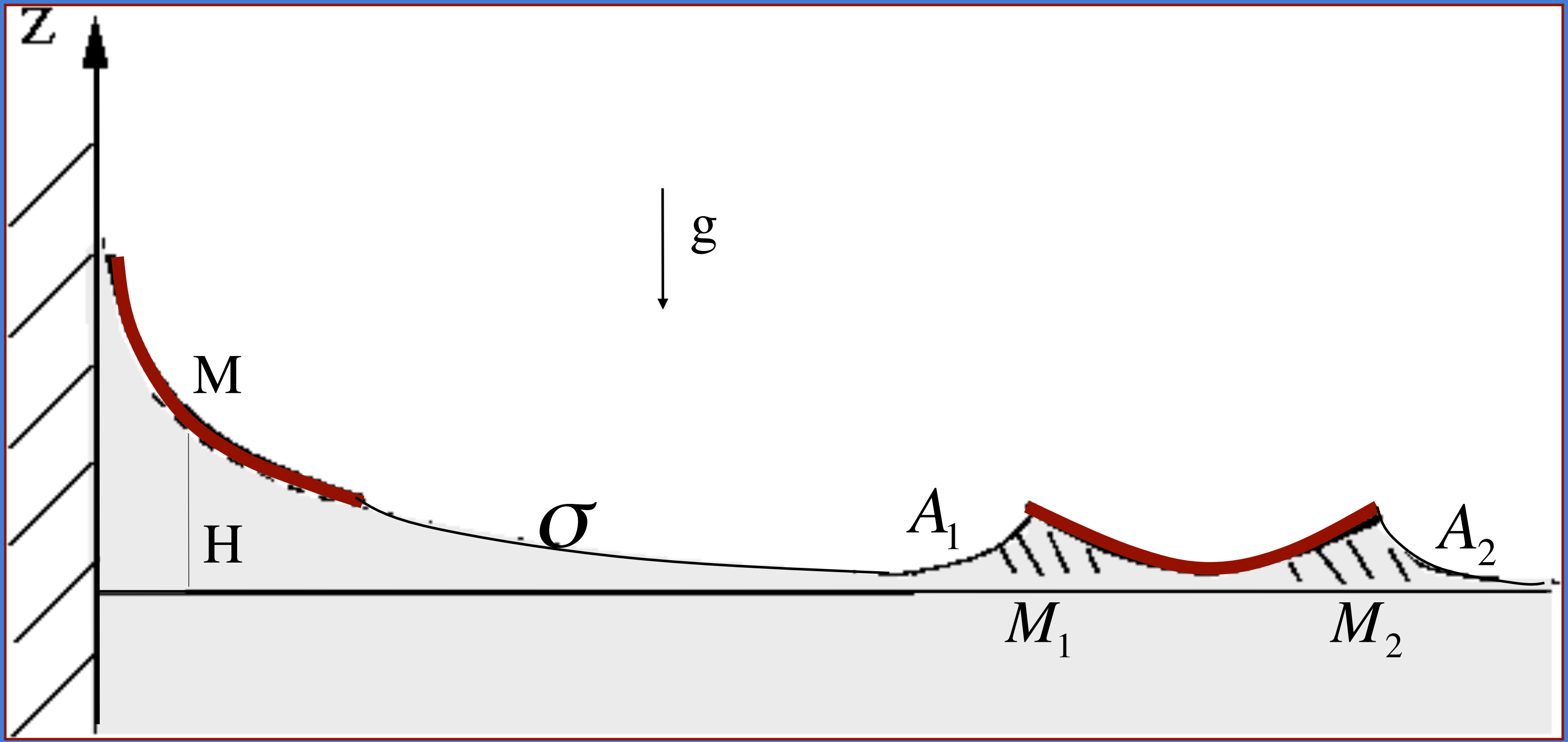
Meniscus climbing

Hu & Bush (Nature, 2005)



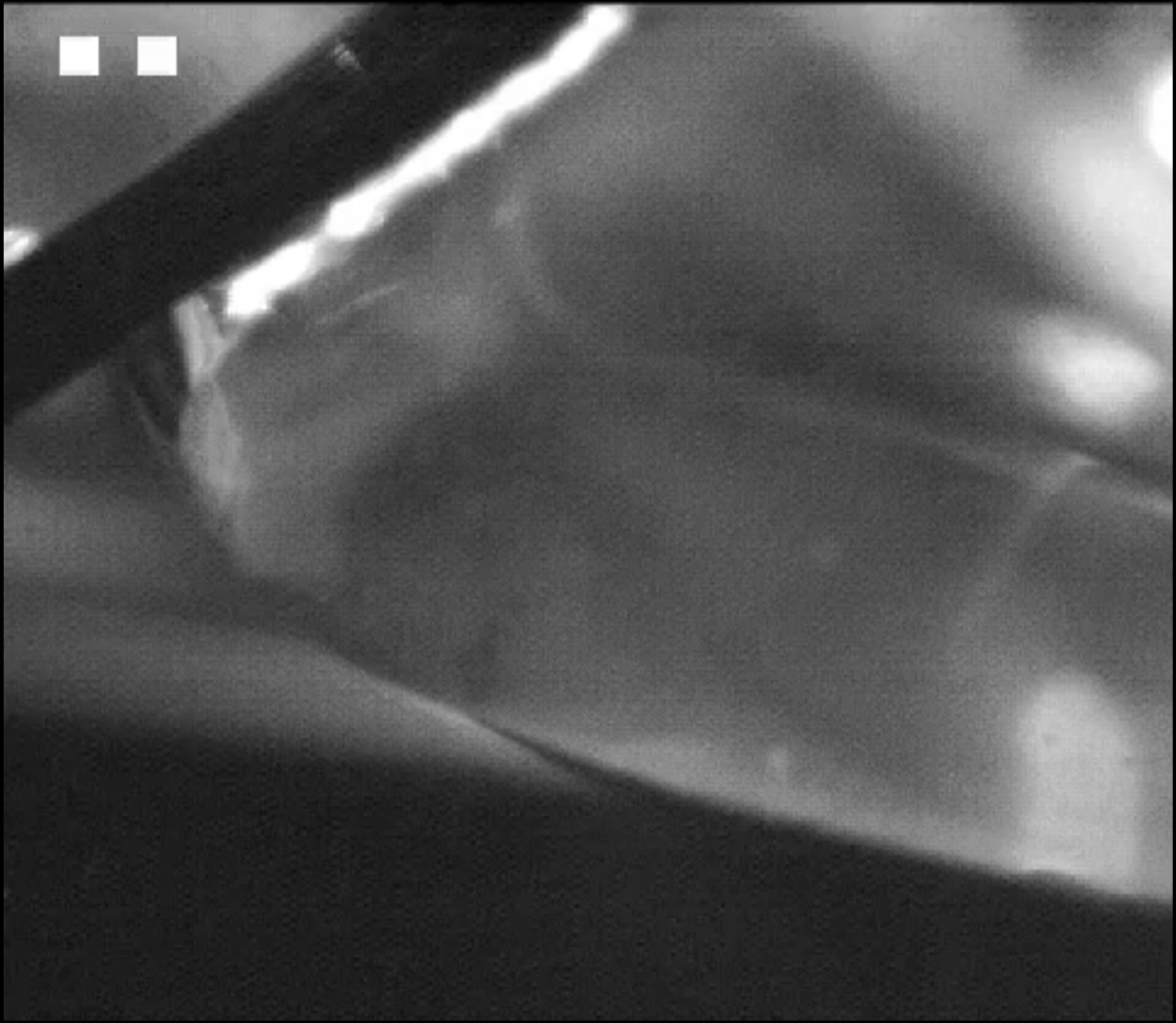
- Anurida arches its back to match curvature of meniscus
- anomalous surface energy exceeds GPE associated with climb

Meniscus-climbing: Energetics

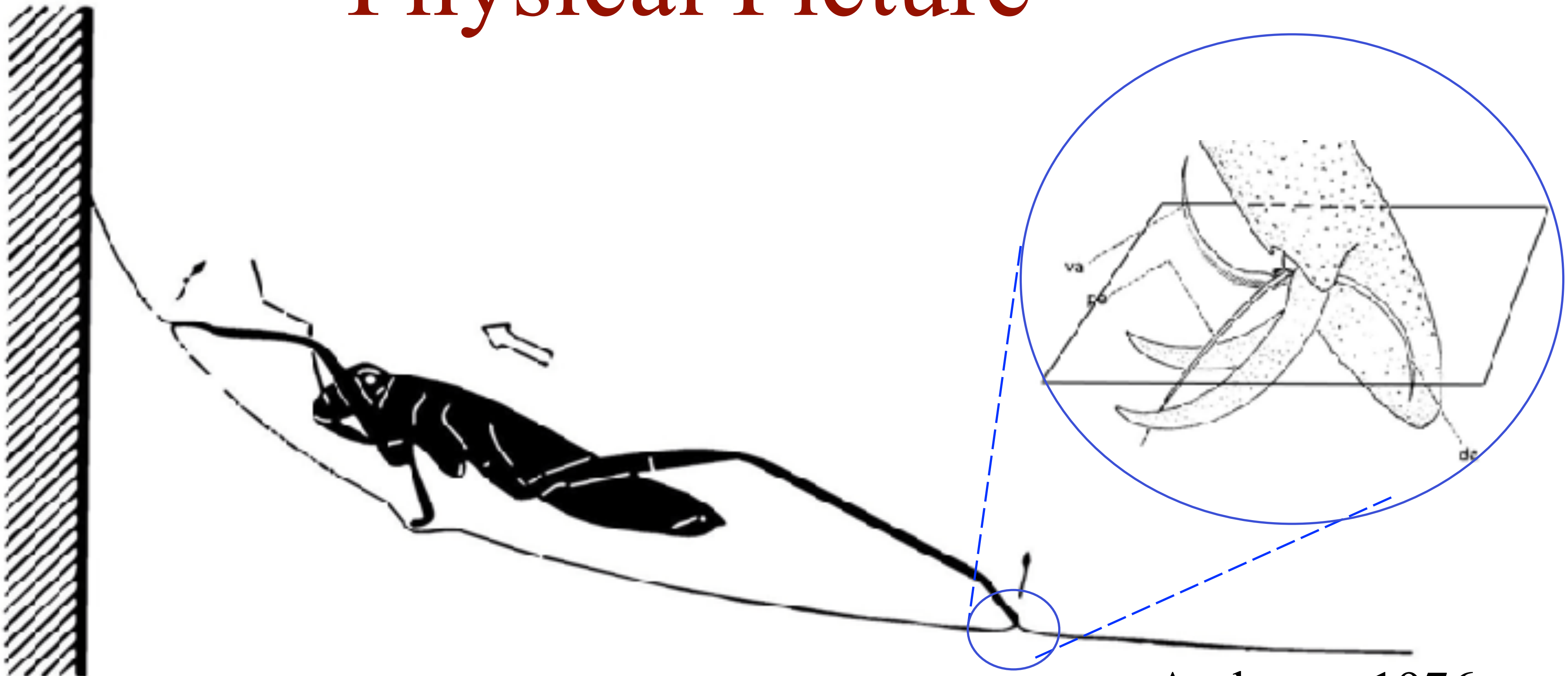


Body climbs provided total energy minimized:

$$\boxed{\sigma(A_1 + A_2) + M_1 g h_1 + M_2 g h_2 > MgH}$$



Physical Picture



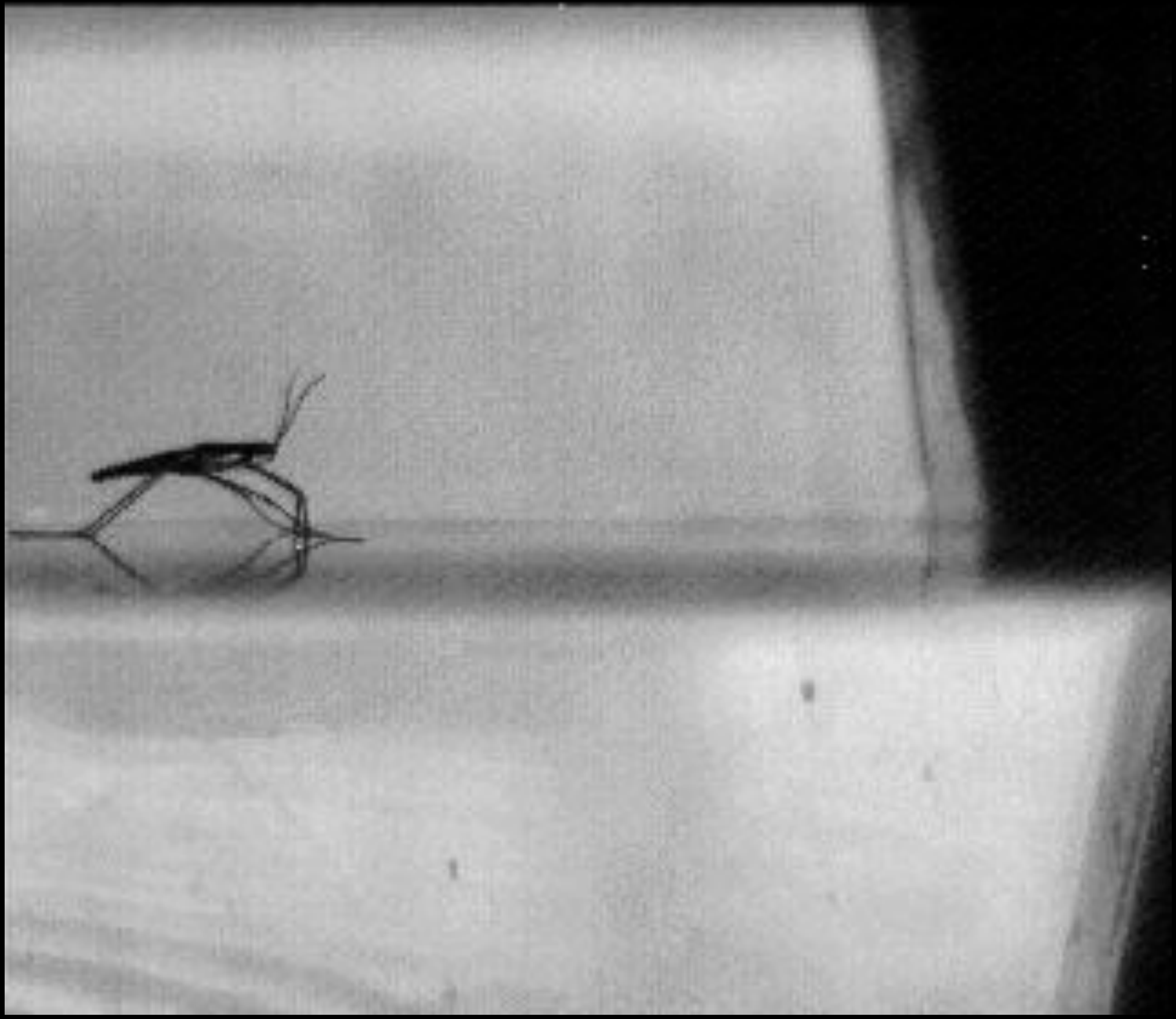
Andersen 1976

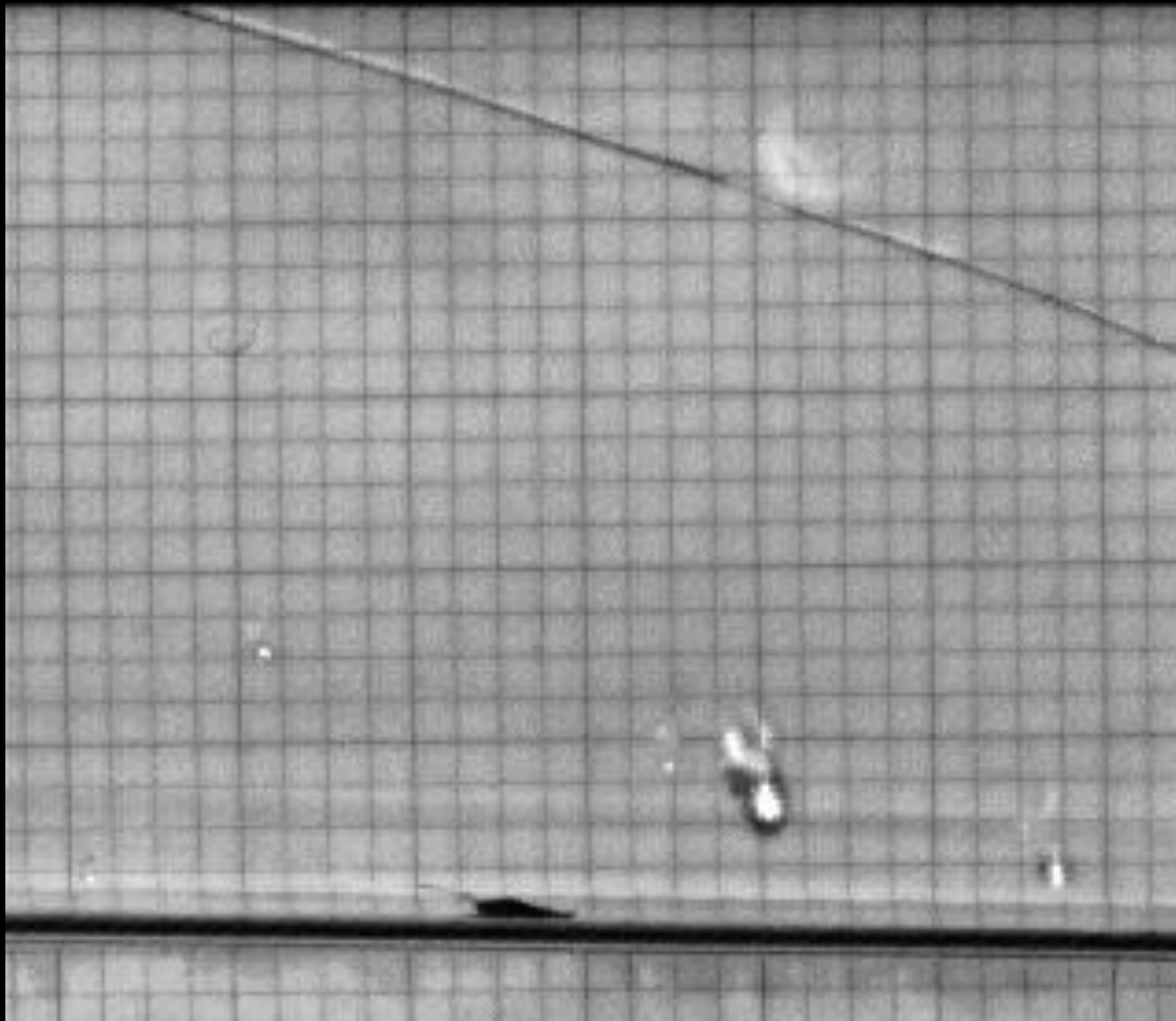
- exploit attraction between like-signed menisci
- pull up with front legs to generate lateral force
- pull up with rear legs to balance torque
- push down with middle legs to support weight

Other uses for capillary attraction

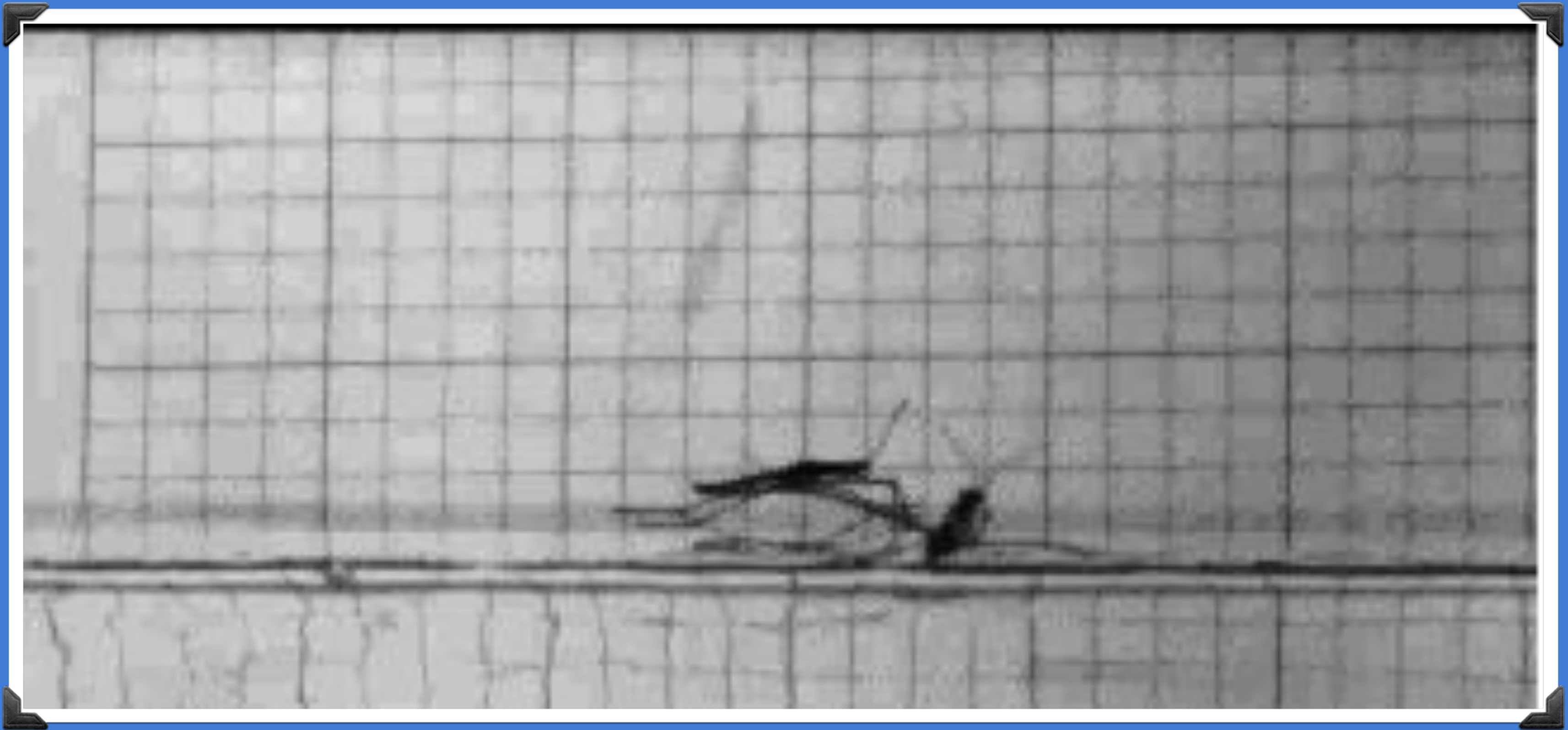
	$\rho g z\ A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n}\ A$	$\underline{\nabla} \sigma\ A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

$\underbrace{\hspace{6em}}_{\text{quasi – static propulsion}}$

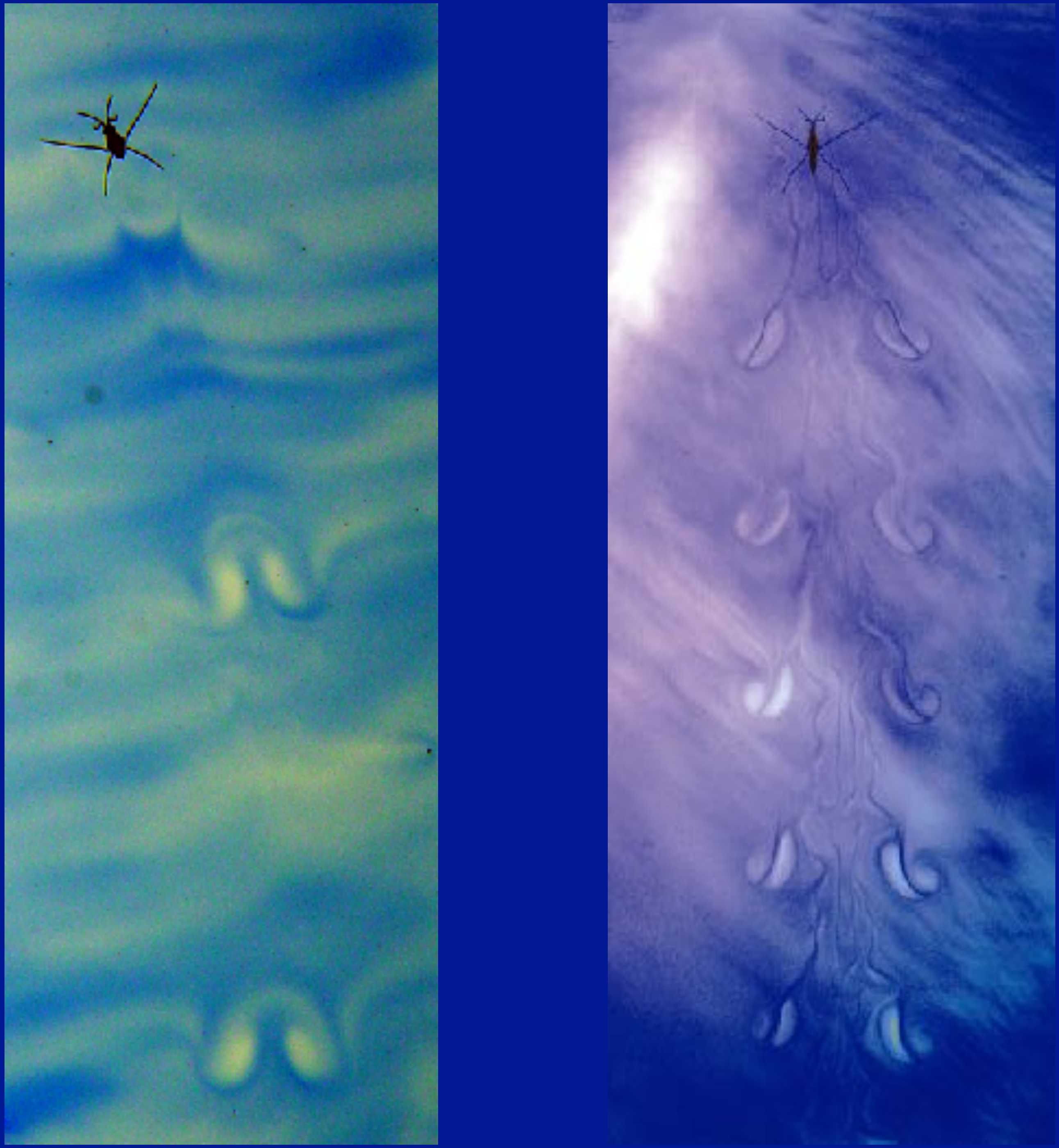


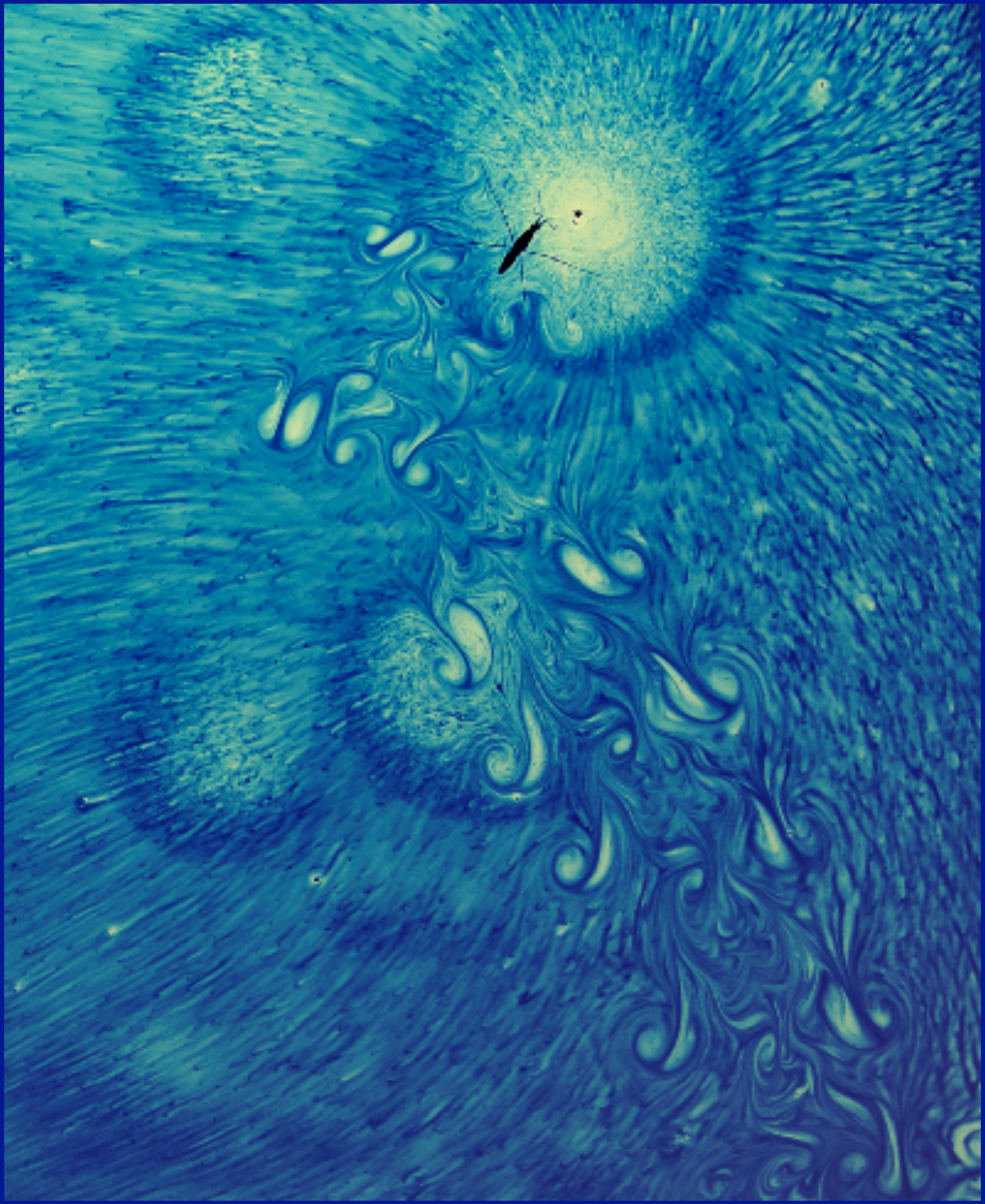


Water strider combat/courtship



Surface tension sets the upper bound on the size of water-walking insects.

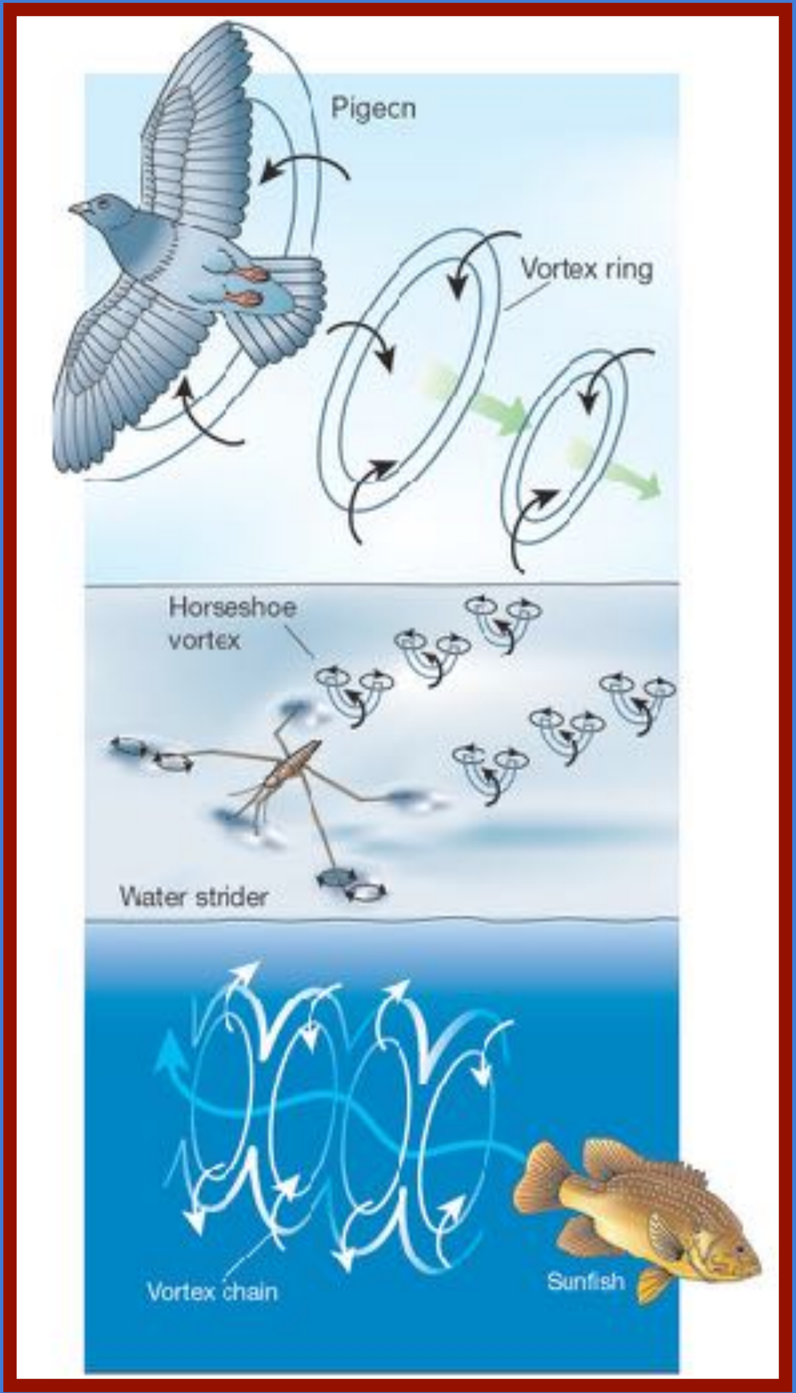




Flying

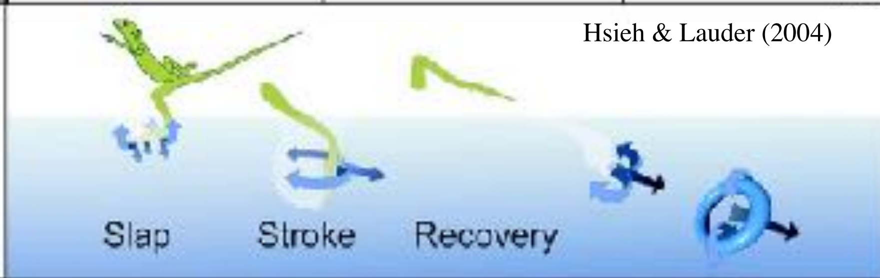
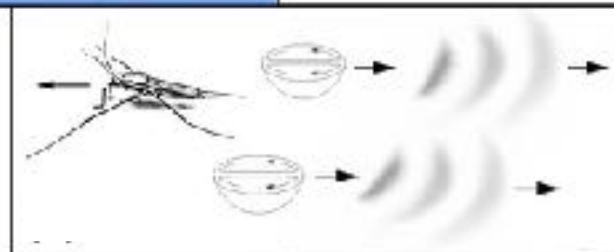
Rowing

Swimming



Dickinson (2003)

SUMMARY

	Buoyancy	Added mass	Inertia	Curvature	Marangoni
Surface slapping			Hsieh & Lauder (2004)		
Rowing & walking					
Meniscus climbing					
Marangoni propulsion					

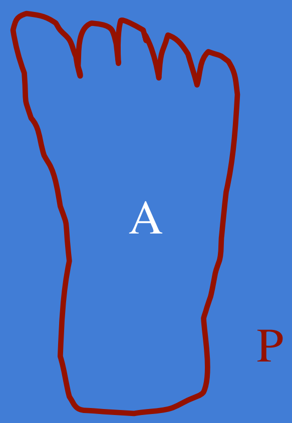
"And when the disciples saw him walking on the sea,
they were troubled, saying, `It is a spirit';
and they cried out for fear."

- Matthew 14:26, King James version

Can man walk on water?

Imagine a man of weight M = 70 kg who can run at U = 10 m/s.

How big must his feet be to walk on water?



Option 1: use surface tension $\sigma = 70$ dynes/cm

Vertical force balance: $$\mathrm{M}\,g = \sigma \mathrm{P}$$

Requires feet with perimeter: $$\mathrm{P} = \frac{\mathrm{M}\,g}{\sigma} \approx 10 \text{ km}$$

Option 2: run via slapping mode

Vertical force balance: $$M\,g = \rho\,U^2 A$$

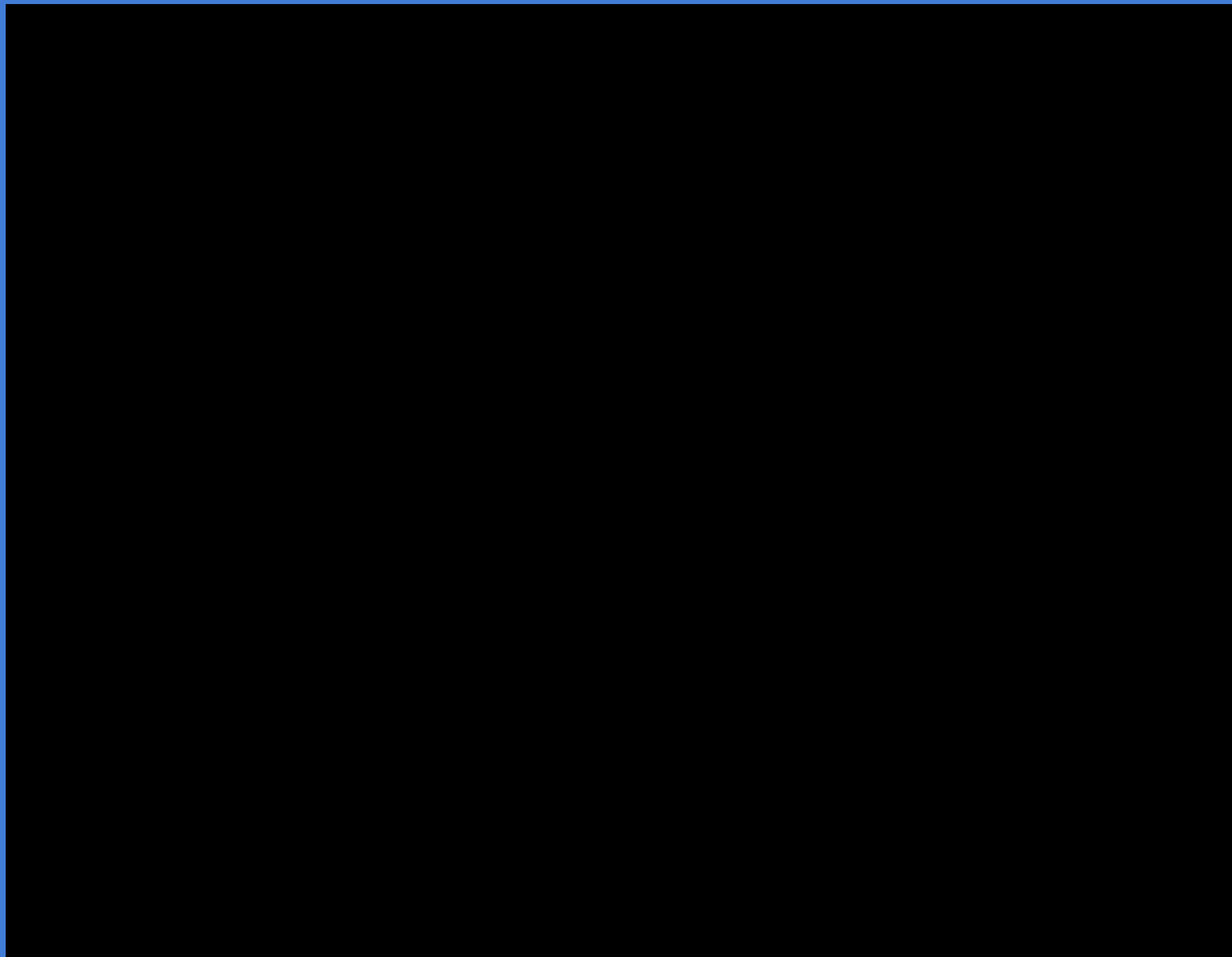
Requires feet with area: $$A \approx 1\,m^2$$

Power requirements: unaided, a man would need to run 30 m/s and generate 15 times as much muscle power (Glasheen & McMahon 1996)

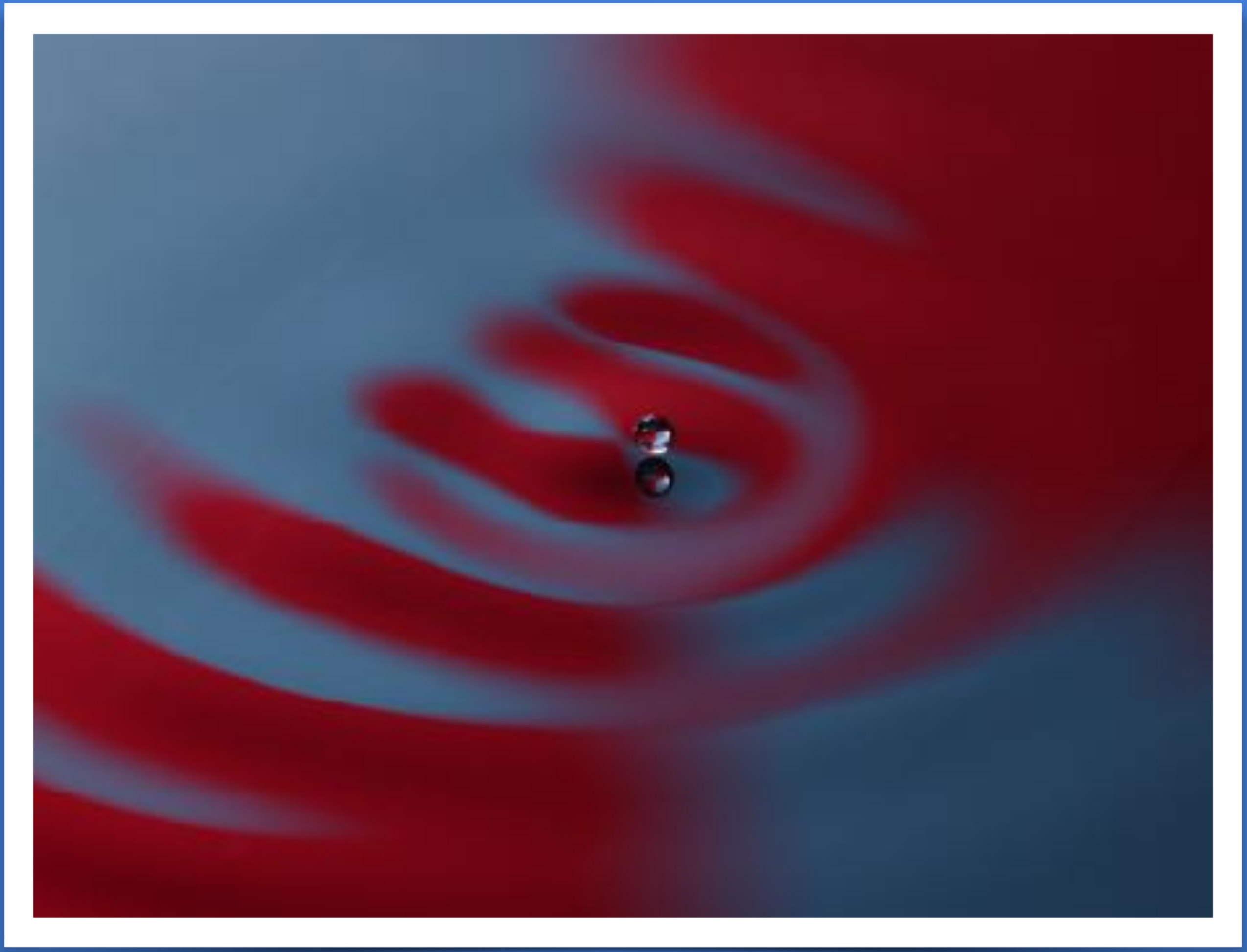
Flotation devices required….

Mizugo Ninja, 12th century

Leonardo da Vinci



Hydrodynamic quantum analogs



A hydrodynamic metaphor

"Light corpuscles generate waves in an Aethereal Medium, just like a stone thrown onto water generates waves. In addition, these corpuscles may be alternately accelerated and retarded by the waves."

- Newton, Opticks (1704)

Quantum mechanics

- a theory that describes the statistics of microscopic particles
- fails to describe particle trajectories — or flatly denies that they exist

The free particle:

$$\boxed{E = \hbar\,\omega \qquad\qquad \mathbf{p} = \hbar\,\mathbf{k}}$$

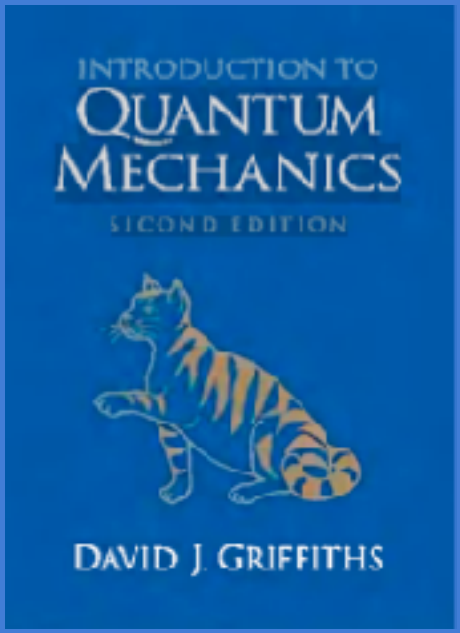
- an association of particle with a wave

But where is the particle, and why does it move?

- an insistence on the completeness of a trajectory-free quantum mechanics has lead to longstanding difficulties

  — the proliferation of quantum interpretations

  — an abundance of paradoxes and troubling language



"It is entirely possible that future generations will look back, from the vantage point of a more sophisticated theory, and wonder how we could have been so gullible."

- D. J. Griffiths

Hydrodynamic quantum analogs

- in 2005, Couder and Fort discovered a hydrodynamic pilot-wave system in which a particle moves in resonance with a guiding wave
- the first macroscopic realization of the double-solution pilot-wave dynamics proposed by Louis de Broglie in the 1920s
- exhibits several features of quantum systems thought to be exclusive to the microscopic, quantum realm

THE QUESTIONS RAISED

What are the key dynamical features responsible for the quantum-like behavior?

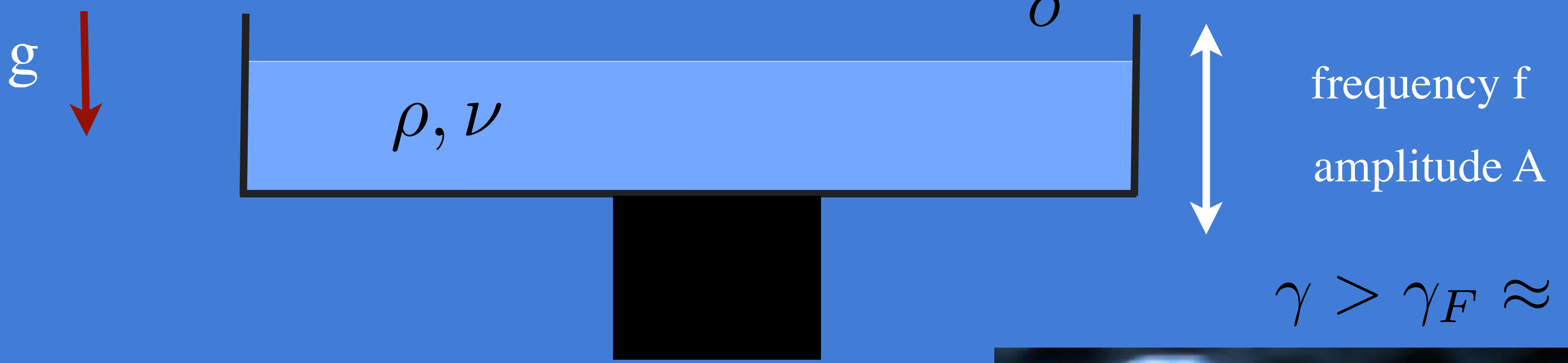
What are the potential and limitations of this hydrodynamic system as a quantum analog?

Can it guide us towards a rational theory for quantum dynamics?

Faraday waves

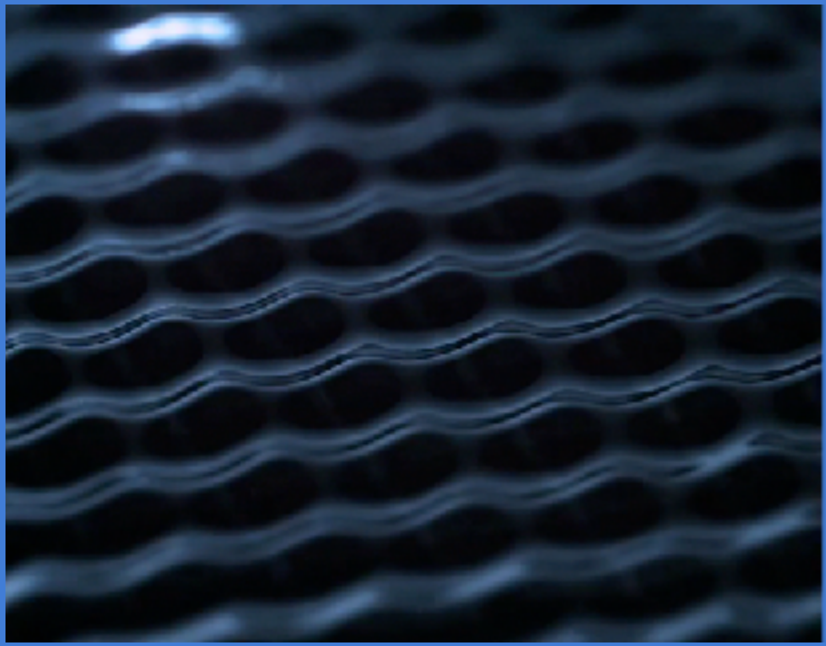
Faraday (1831)

- surface undulations with twice the forcing period, a parametric instability
- arise above a threshold γ_F that depends on fluid depth, viscosity, surface tension



$$\gamma > \gamma_F \approx 4g$$

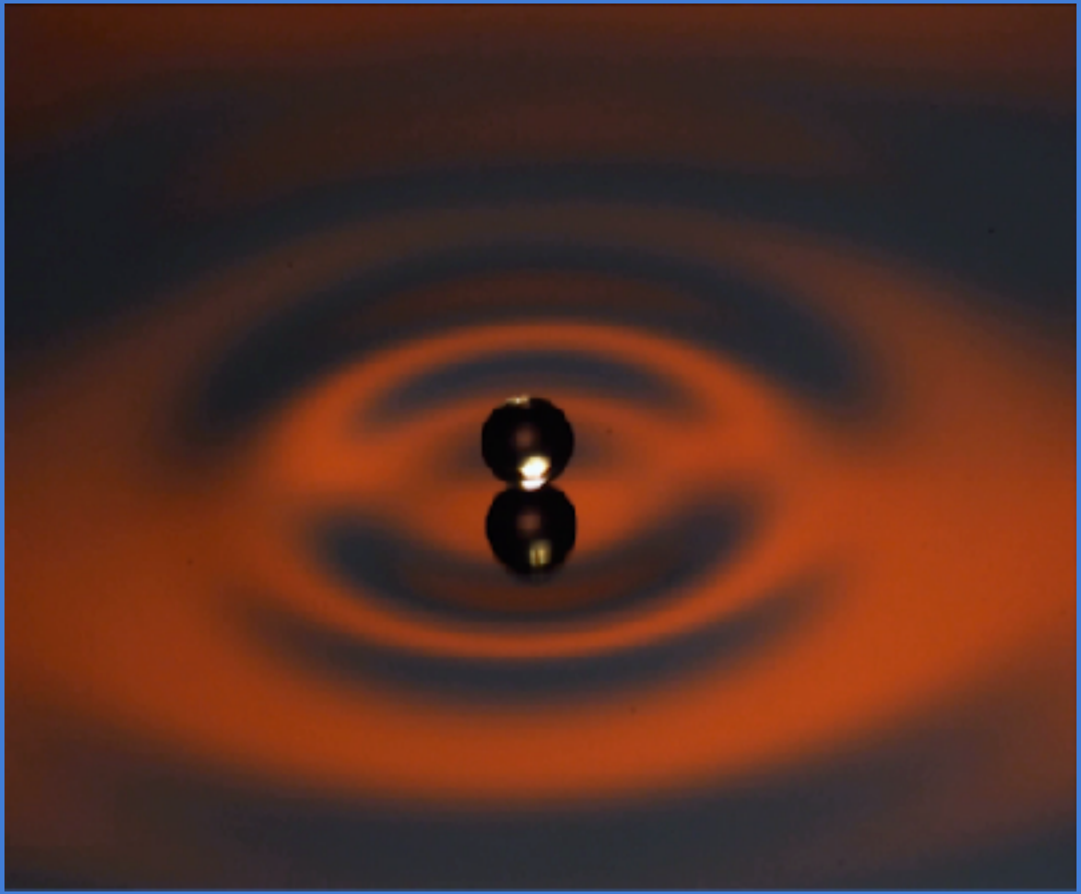
Vibrational acceleration: $\gamma = A(2\pi f)^2$



Noncoalescence on a vibrated fluid bath

Jearl Walker (1978), Protière et al. (2005)

- coalescence avoided provided impact time is less than time required for air layer between drop and bath to drain to ~100 nm



f ~ 50 Hz

50cS
Si oil

- owing to surface tension, interface behaves effectively as a linear spring

Gilet & Bush (2009ab)

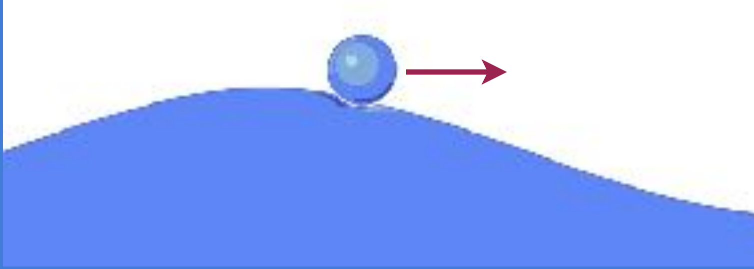
The Couder walker

Couder et al. (2005)

- ***resonance condition***: drop bounces at Faraday frequency
- resonant bouncing droplets may be destabilized by their wave field, walk
- spatially extended *walkers* consist of both droplet and guiding/pilot wave
- dynamics is ***non-Markovian, hereditary***: wave force depends on walker's history
- proximity to Faraday threshold prescribes longevity of waves, **`path memory'**

Eddi et al. (2011)

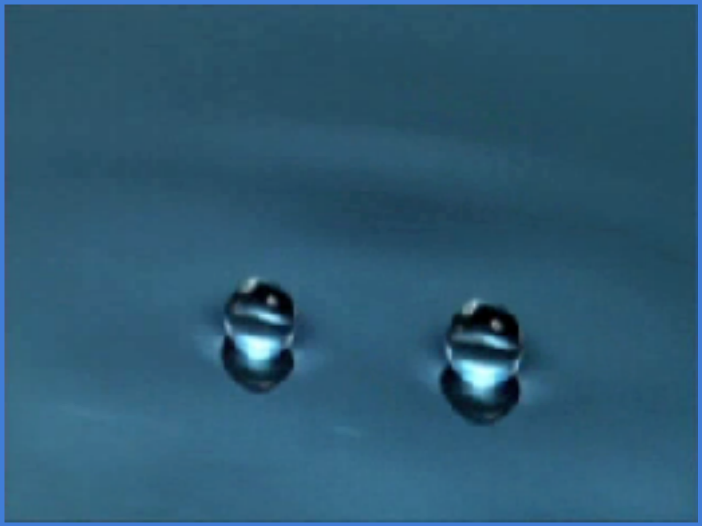
Strobed pilot-wave dynamics

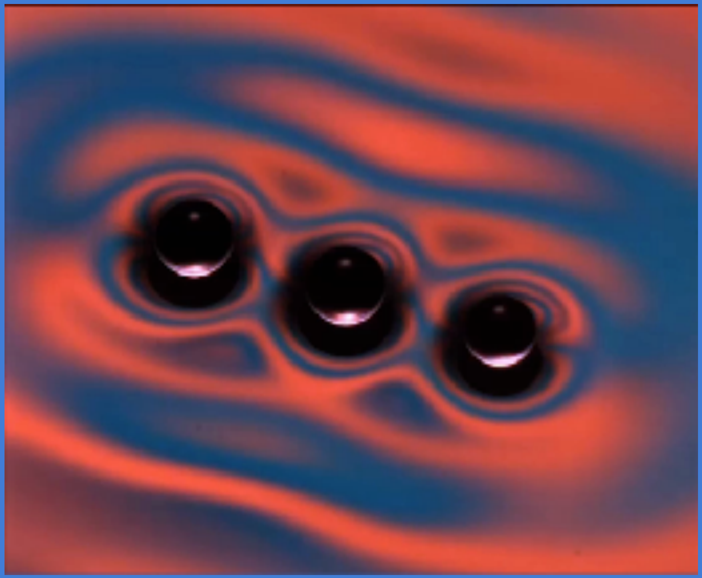


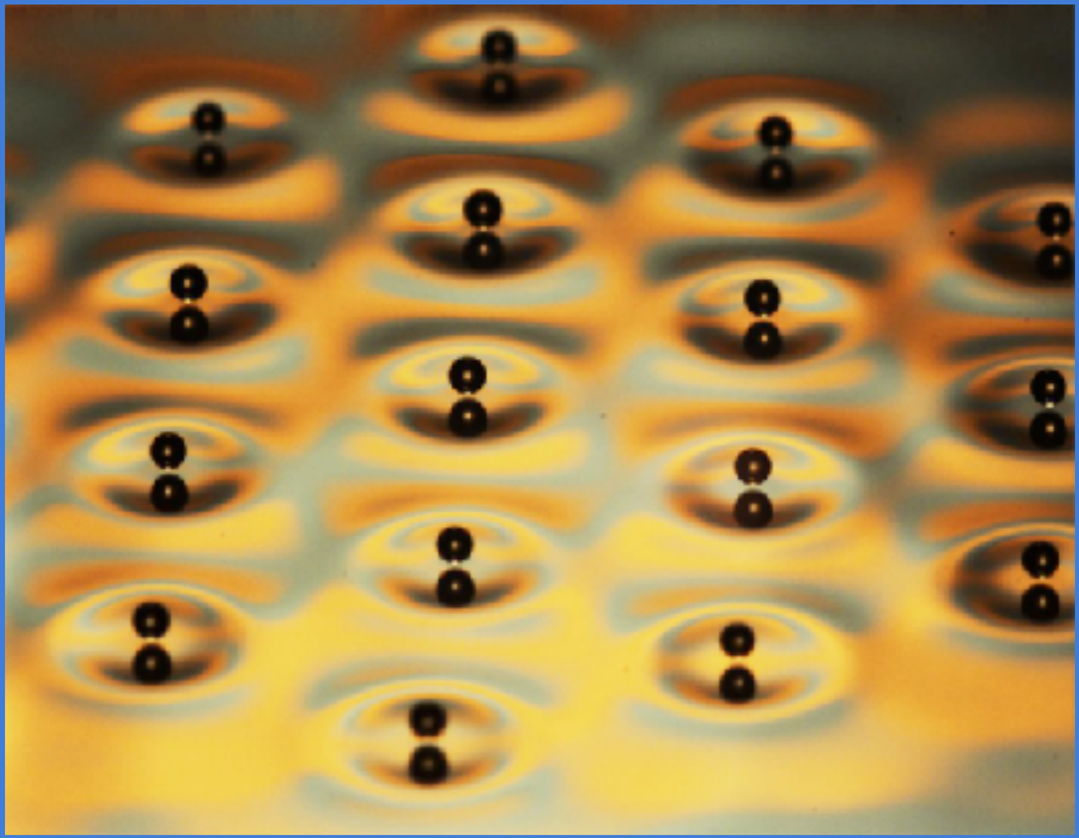
- strobe the system once per bounce cycle
- conceals the vertical dynamics responsible for the guiding wave
- drop appears to surf on the interface, dressed by a quasi-monochromatic pilot-wave field that is stationary in the drop's frame of reference

Static bound states

- multiple droplets lock into phase, interact through their wave fields







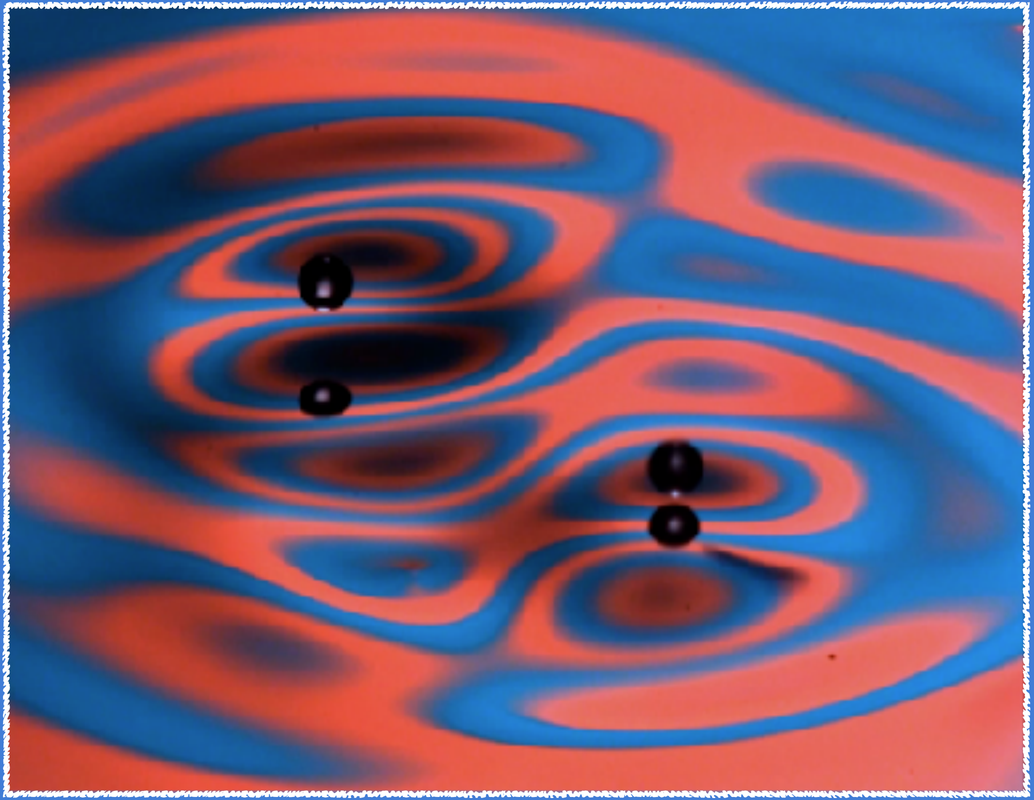
- bond lengths quantized by wave field

Eddi et al. (2009), Couchman et al. (2019)

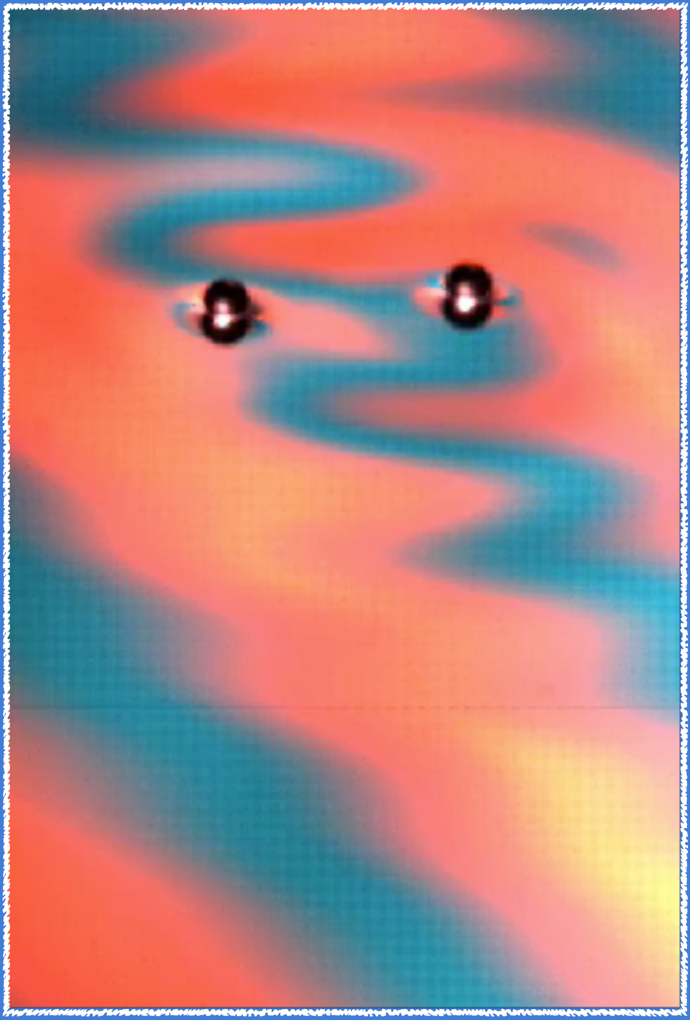
Dynamic bound states

- bond lengths quantized by wave field

Orbiting pairs

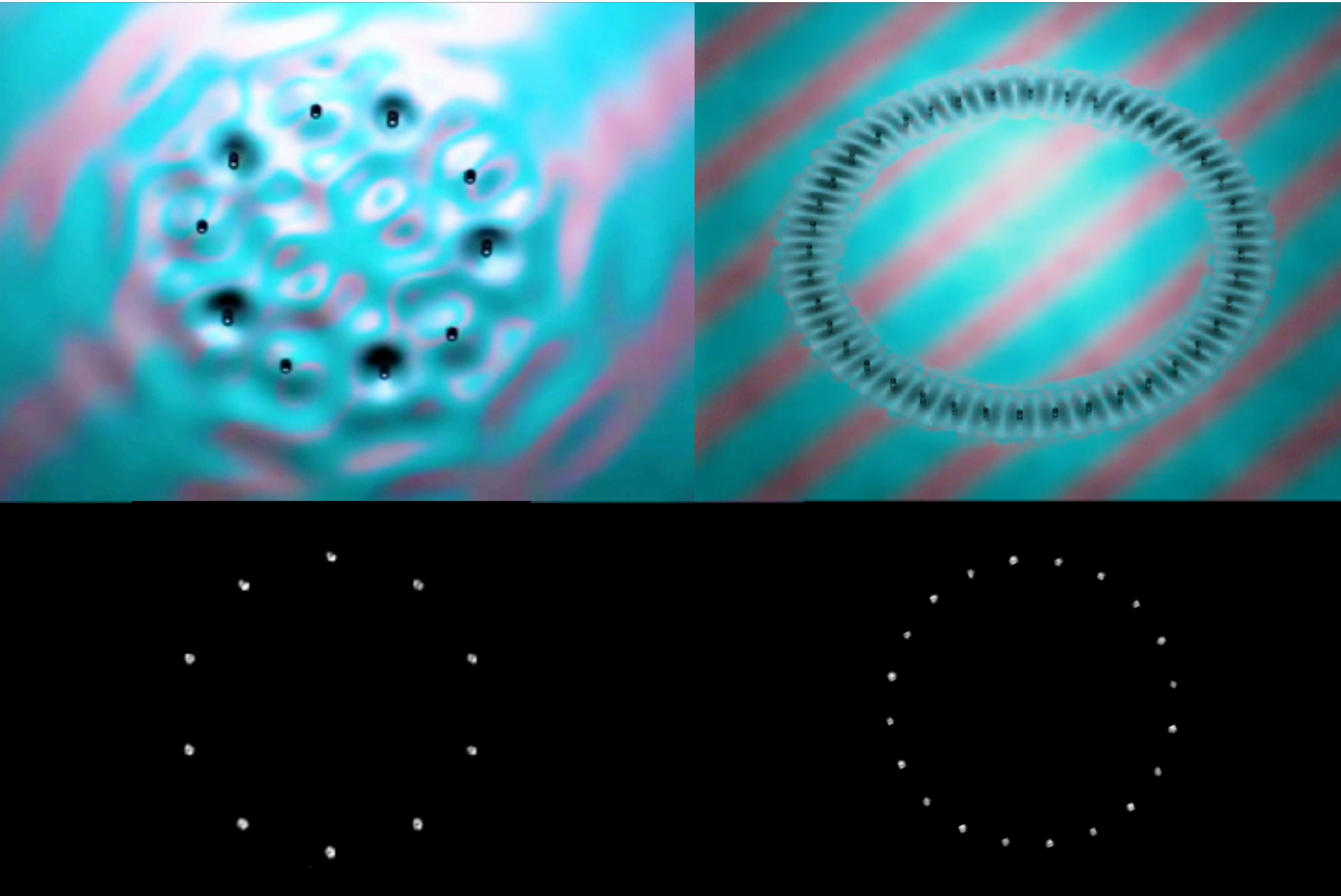


Promenading pairs



Rings of bouncing droplets

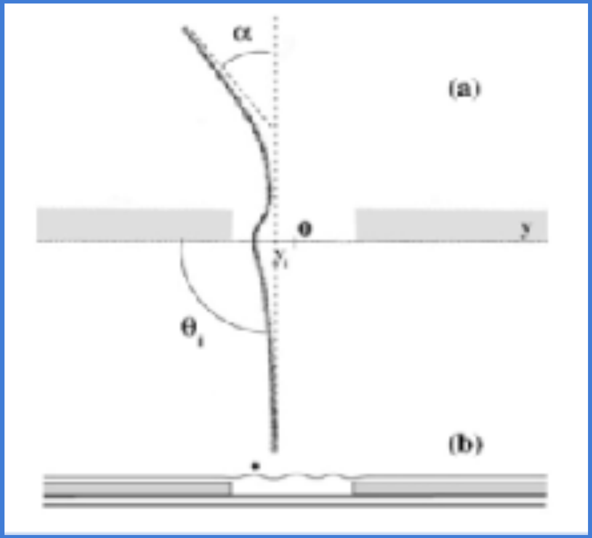
Couchman & Bush (2020)
Thomson, Couchman & Bush (2020)

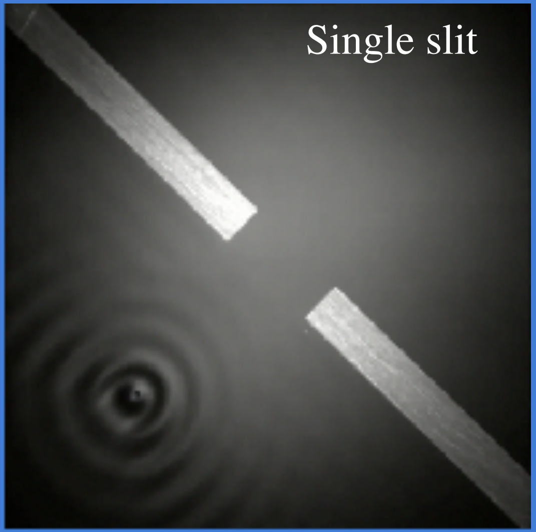


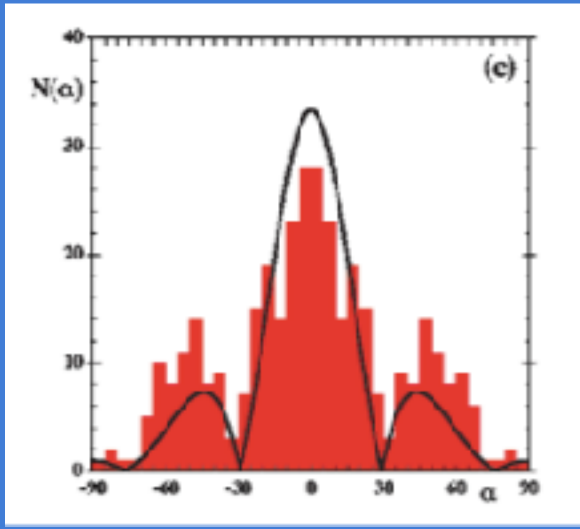
Quantum mechanics writ large

Single-particle diffraction and interference

Couder & Fort (2005)

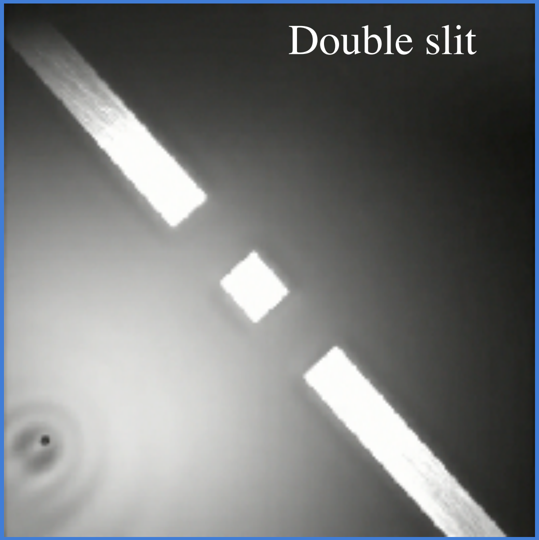


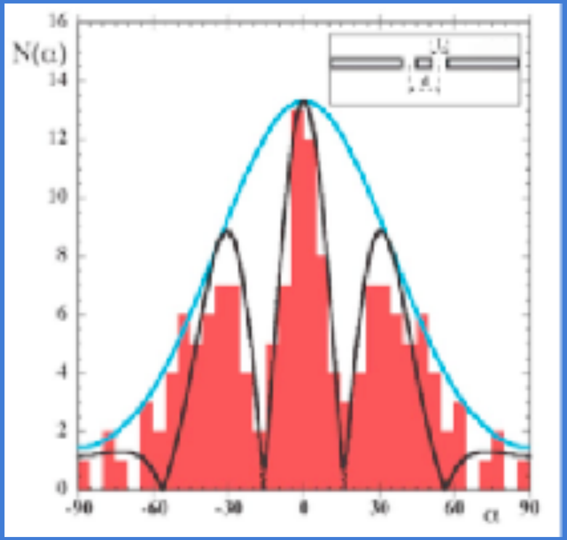




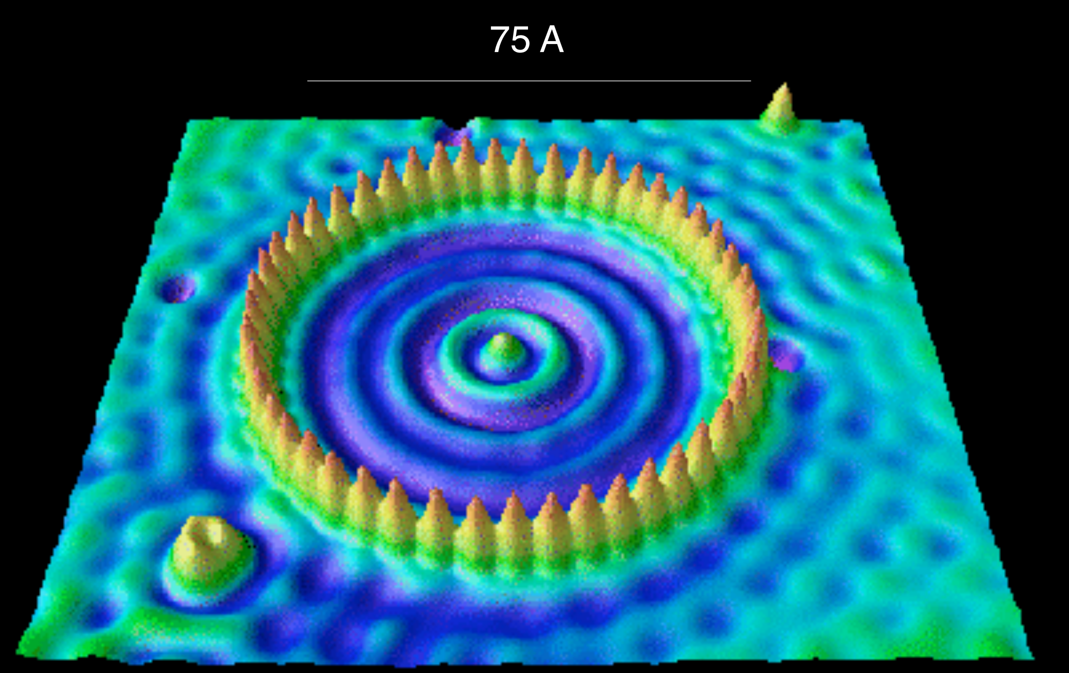
" A phenomenon which is impossible, absolutely impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery."

- Richard Feynman





- **coherent, wave-like statistics emerge from chaotic pilot-wave dynamics**
- results contested by Bohr & coworkers (2015, 2016), revisited by Pucci et al. (2017)

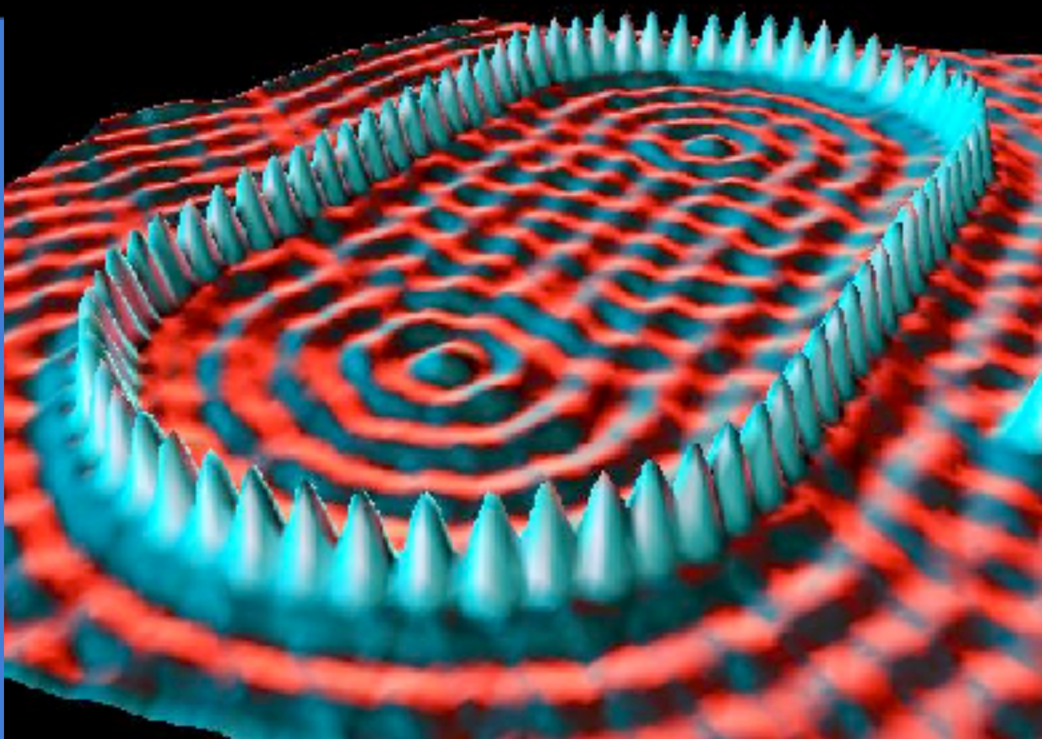


The quantum corral

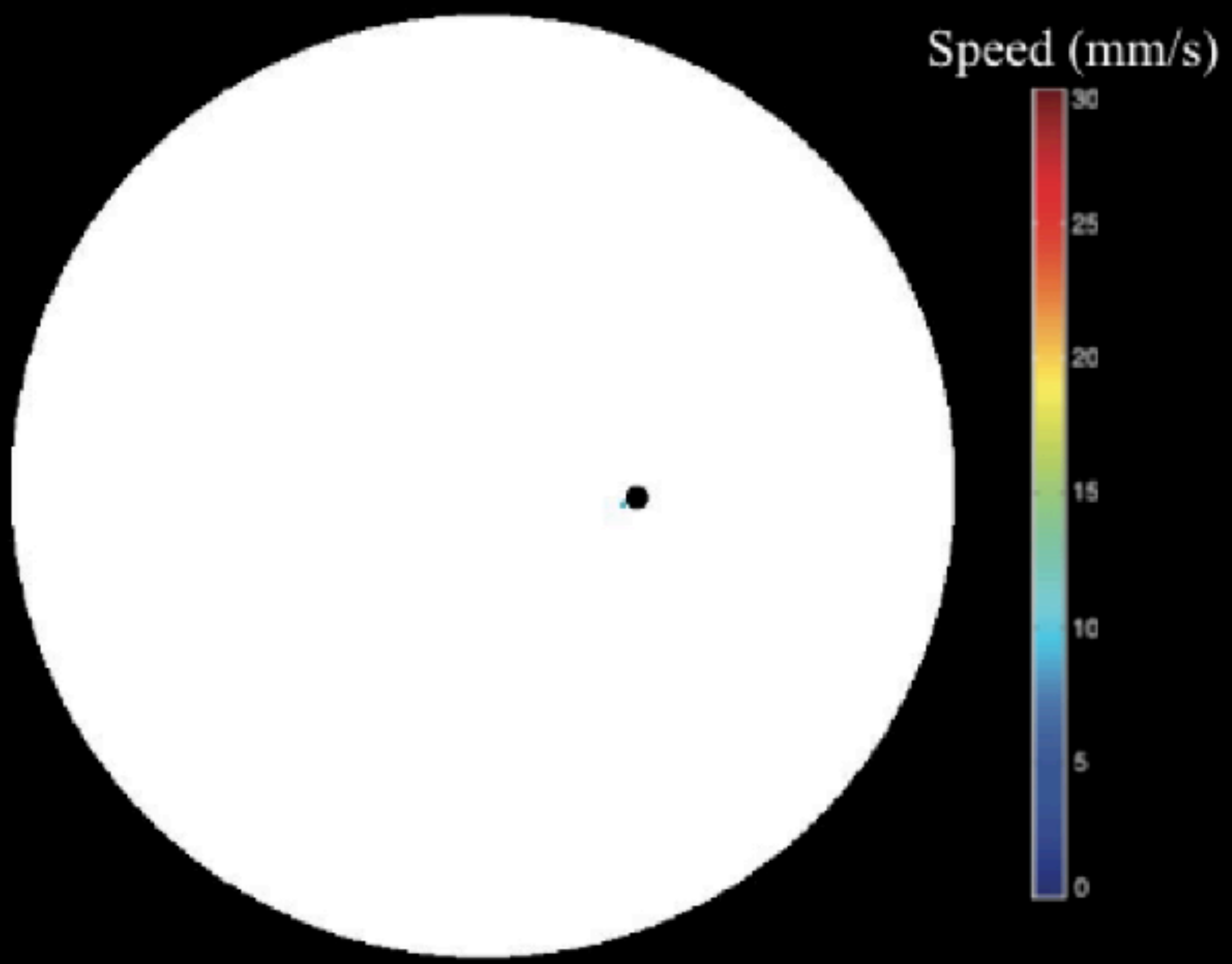
Crommie, Lutz & Eigler (1993)

Fiete & Heller (2003)

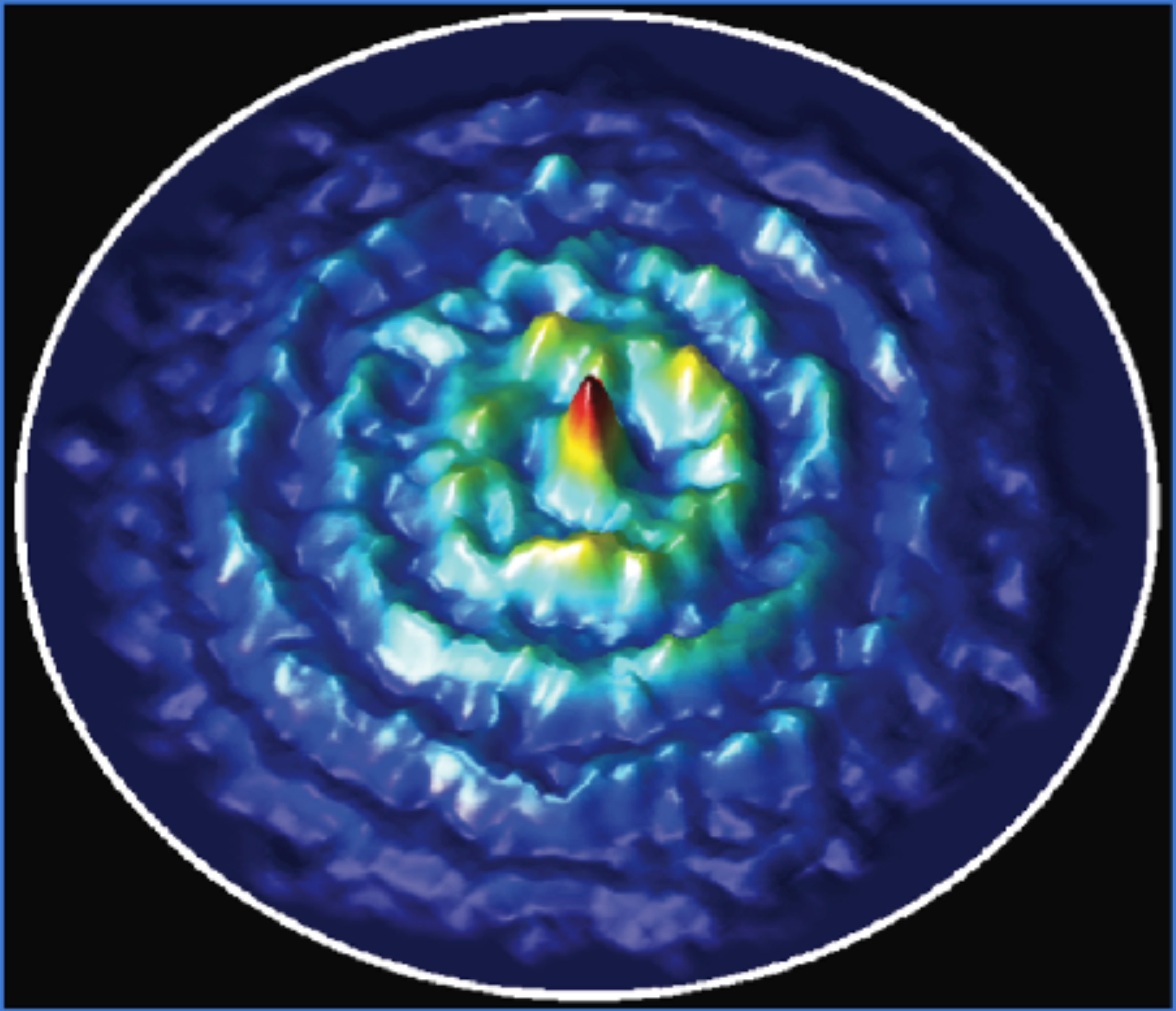
- de Broglie waves evident in the pdf of a sea of electrons trapped on a metal surface, excited by an SEM



Droplet walking in a circular corral



Probability density function



Harris, Moukhtar, Fort, Couder & Bush (2013)

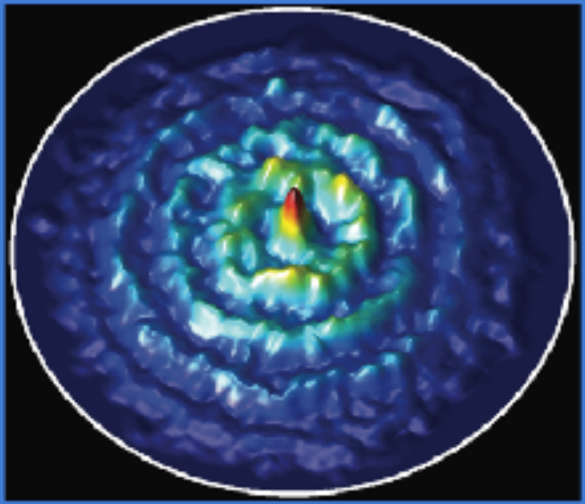
- **pdf similar to the amplitude of the Faraday wave mode of the cavity**
- **coherent, wave-like statistics emerge from chaotic pilot-wave dynamics**

Emerging physical picture: 3 time scales

- **fast** dynamics: bouncing at resonance creates monochromatic wave field

- **intermediate** (strobed) pilot-wave dynamics: droplet rides its instantaneous guiding wave

- **long-term statistical** behaviour described by Faraday wave modes



De Broglie's relativistic pilot-wave theory

- an attempt to reconcile relativity and QM through consideration of the wave nature of matter

Einstein-de Broglie relation:

$$\boxed{\begin{aligned} E &= m\,c^2 \\ E &= \hbar\,\omega \end{aligned}}$$

RELATIVITY

QM

Natural frequency →

$$\boxed{\omega_c = \frac{mc^2}{\hbar}}$$

Compton frequency

But what is happening at this frequency?

- de Broglie suggested an exchange between rest mass energy and field energy
- in modern QFT, this sets the time and length scale of particle pair production

de Broglie's pilot-wave theory: The double-wave solution

" A freely moving body follows a trajectory that is orthogonal to the surfaces of an associated wave guide".

- Louis de Broglie (1892-1987)

- Ψ is the probability wave, as prescribed by standard quantum theory
- $\phi = |\phi| \; e^{i\Phi/\hbar}$ is a real physical wave responsible for guiding the particle
- wave generated by internal particle vibration (*Zitterbewegung*) at the Compton frequency: $\omega_c = \dfrac{m_0 c^2}{\hbar}$

- a solution of Klein-Gordon equation triggered by oscillations in rest mass
- particle follows point of constant wave amplitude: his guidance equation yields

  $\mathbf{p} = \gamma m_0 \mathbf{x_p} = \nabla\Phi = \hbar\mathbf{k}$ for a monochromatic wave $\Phi = \mathbf{k}\cdot\mathbf{x} - \omega t$

- **Harmony of Phases**: the particle oscillates in resonance with its guiding wave
- wave generation mechanism, form of ϕ not specified

de Broglie's pilot-wave theory

- **fast** dynamics: mass oscillations at

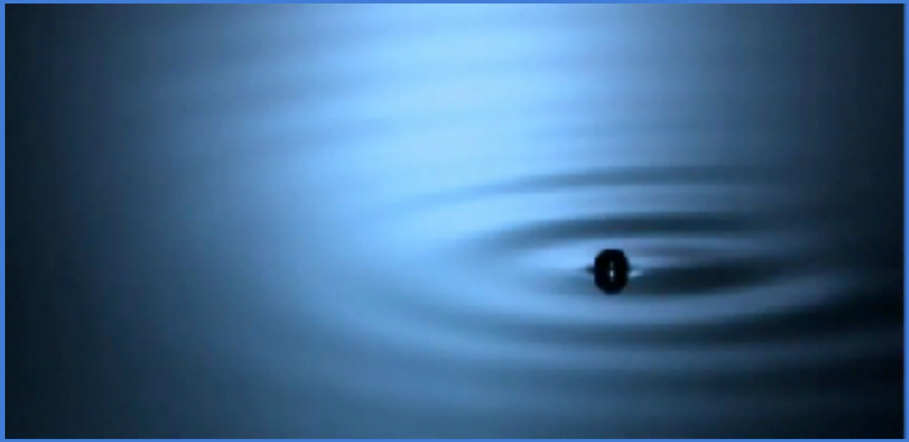
$$\omega_c = \frac{m_0 c^2}{\hbar} \quad \text{create wave field}$$

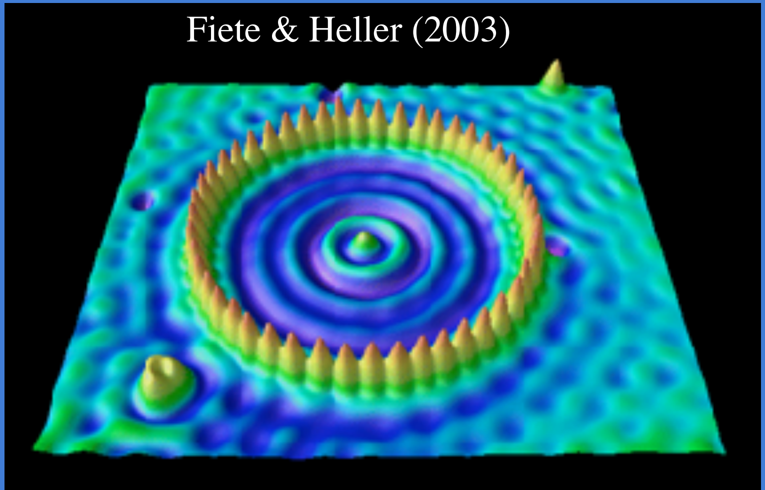
centered on particle

- **intermediate** pilot-wave dynamics: particle rides its guiding wave field such that

$$\mathbf{p} = \hbar\, \mathbf{k}$$

- **long-term statistical** behaviour described by standard quantum theory



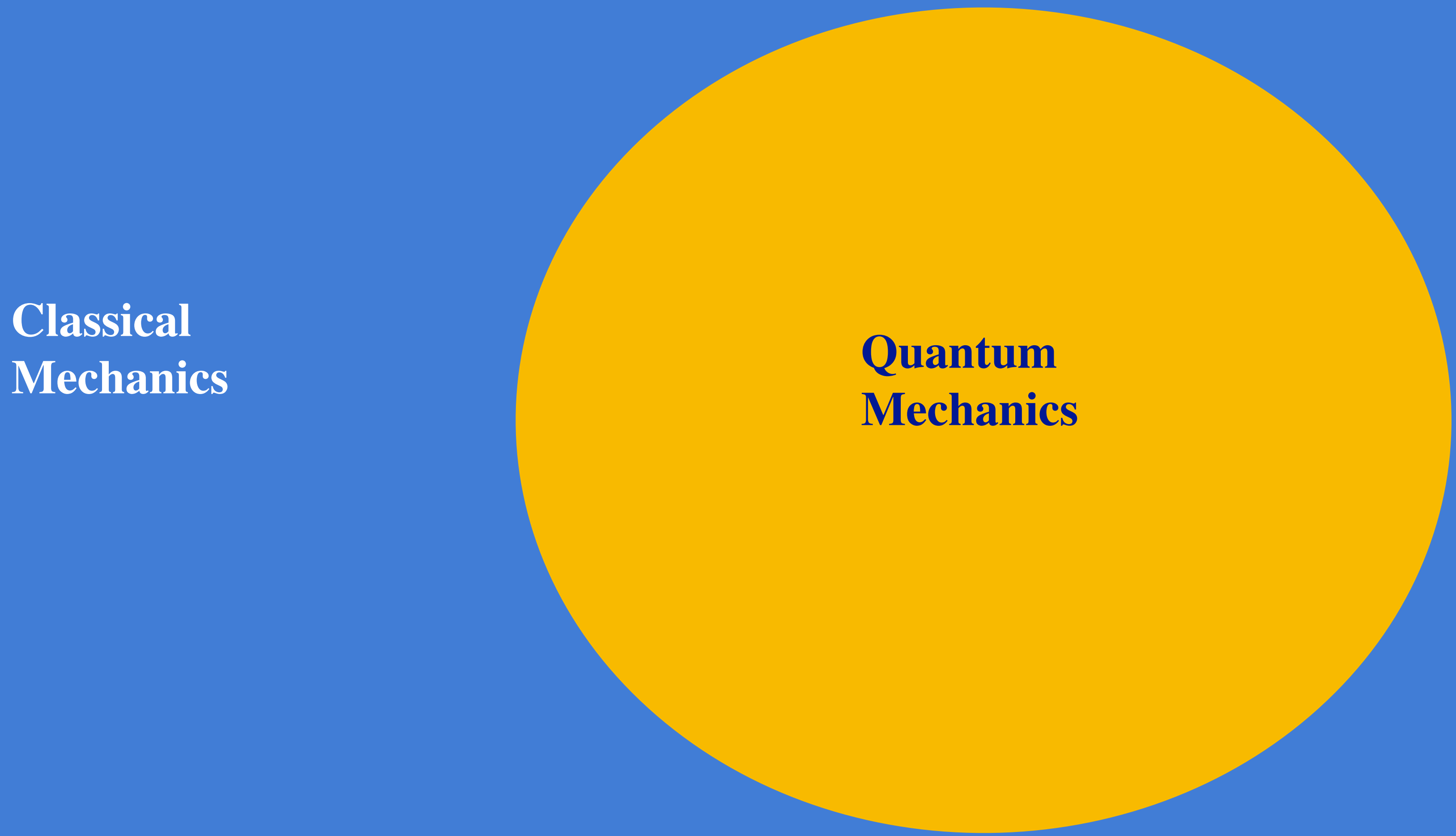


Fiete & Heller (2003)

	de Broglie	Walkers
WAVE TRIGGER	ZITTERBEWEGUNG	Bouncing
VIBRATION FREQUENCY	$\omega_c = \frac{m_o c^2}{\hbar}$	$\omega_d = \sqrt{\frac{\sigma}{\rho a^3}}$
WAVES	Matter waves / Higgs field	Capillary Faraday
WAVE-PARTICLE RESONANCE	Harmony of phases	$\omega_d = \omega_F$
WAVE ENERGETICS	$mc^2 \longleftrightarrow \hbar\omega$	$MgH \longleftrightarrow$ Surface Energy
KEY PARAMETER	$\hbar$	σ
STATISTICAL WAVELENGTH	λ_B	λ_F
VIBRATION LENGTH	$\lambda_c = h/mc$	STEP SIZE

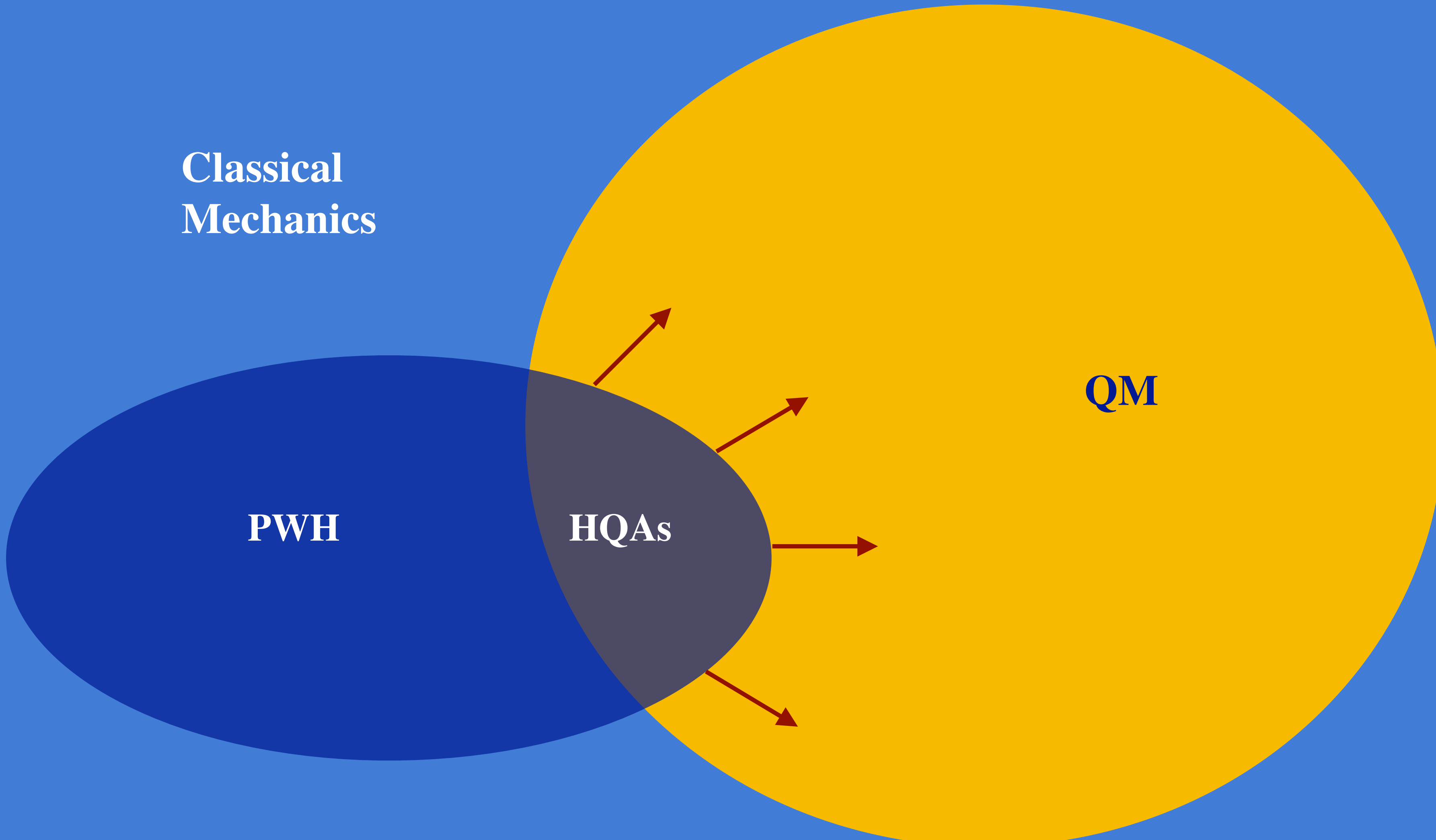
BIG PICTURE

- the landscape before PWH: classical mechanics and quantum mechanics



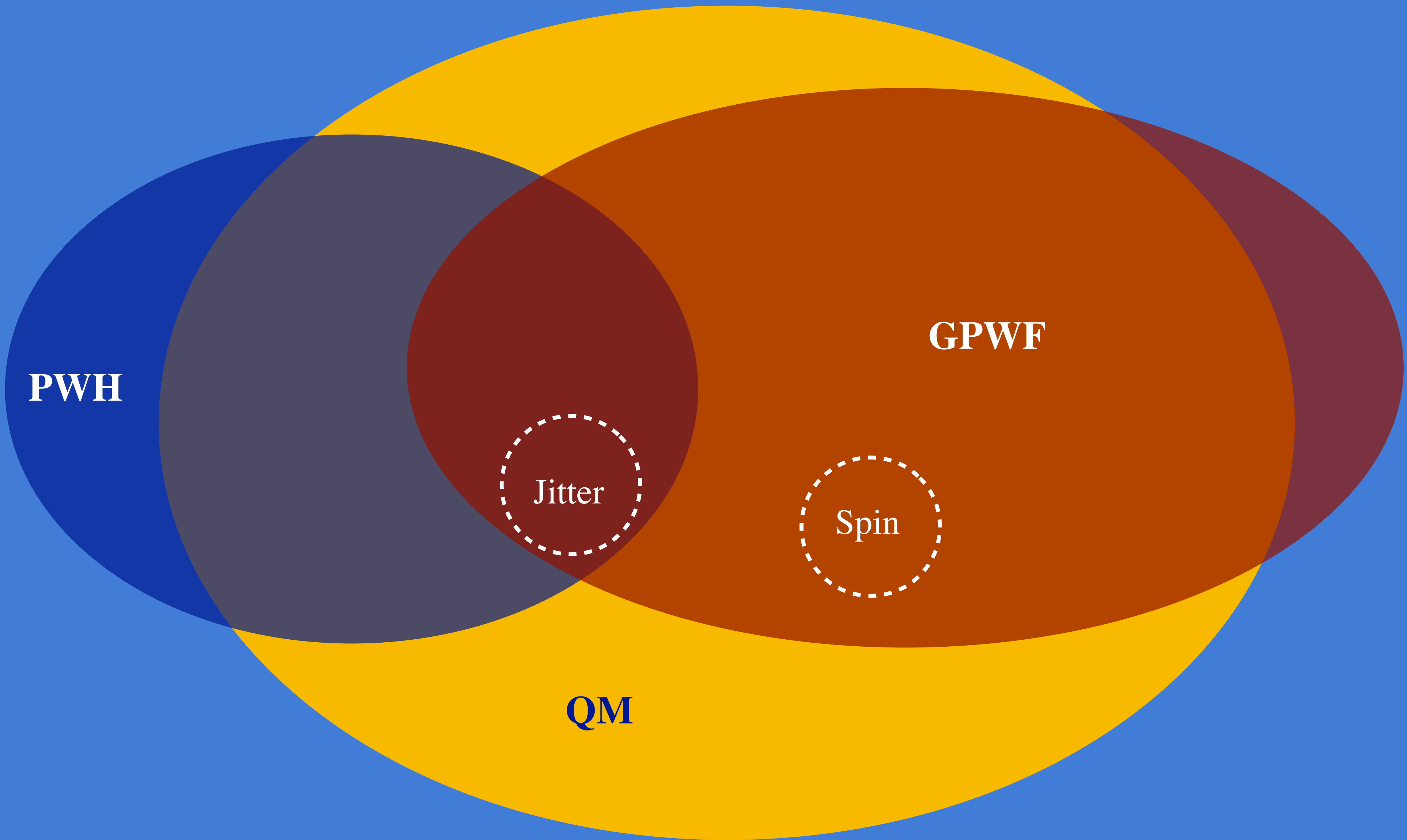
BIG PICTURE

- enter pilot-wave hydrodynamics

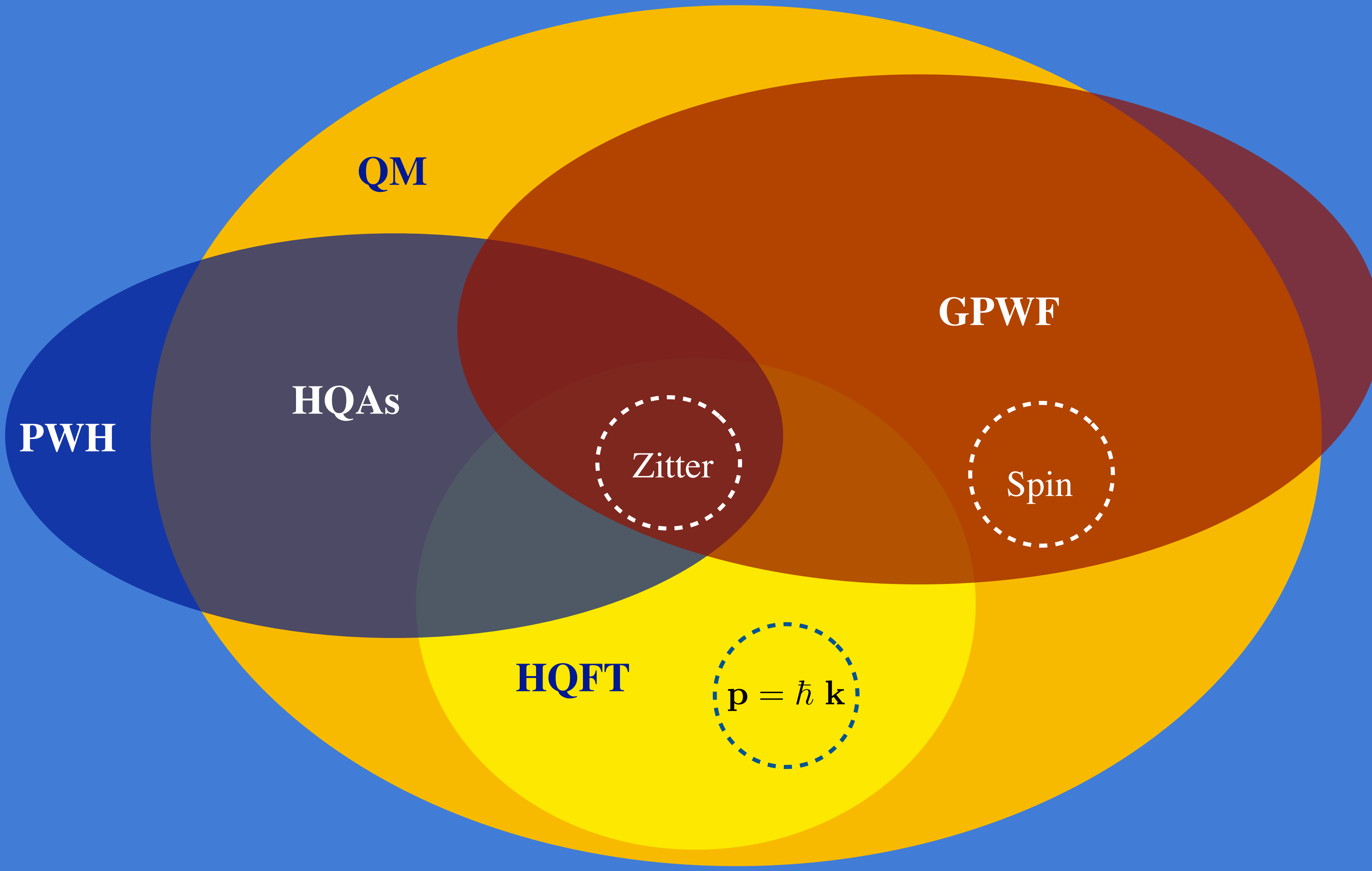


BIG PICTURE

- enter the Generalized Pilot-wave Framework



BIG PICTURE



Wave-particle duality

"Both matter and radiation possess a remarkable duality of character, as they sometimes exhibit the properties of waves, at other times those of particles.

Now it is obvious that a thing cannot be a form of wave motion and composed of particles at the same time - the two concepts are too different."

— *Heisenberg, On Quantum Mechanics, 1930*

Thanks!

“I am, in fact, rather firmly convinced that the essentially statistical nature of contemporary quantum theory is solely to be ascribed to the fact that it operates with an incomplete description of physical systems.”

“In a complete physical description, the statistical quantum theory would take an approximately analogous position to statistical mechanics within the framework of classical mechanics.”

- **Albert Einstein**