Problem 1 Let $\omega_1, \omega_2 \in \mathbb{C} \setminus \{0\}$ be two complex non-zero numbers such that $\frac{\omega_1}{\omega_2} \notin \mathbb{R}$. Show that

$$\sum_{(m,n)\in\mathbb{Z}^2\setminus(0,0)} \frac{1}{(m\omega_1 + n\omega_2)^s}$$

converges for any integers $s \geq 3$.

Problem 2 Let $\Lambda = \{\omega = m\omega_1 + n\omega_2, (m,n)\in\mathbb{Z}^2\} \subset \mathbb{C}$ be a lattice. The Weierstrass function is defined to be

$$\mathfrak{p}(z, \omega_1, \omega_2) = \frac{1}{z^2} + \sum_{\omega\in\Lambda,\omega\neq 0} \left(\frac{1}{(z-\omega)^2} - \frac{1}{\omega^2}\right).$$

Prove the indefinite integral

$$\int \mathfrak{p}(z)^2 dz = \frac{1}{6}\mathfrak{p}'(z) + \frac{1}{12} g_2 \cdot z + C,$$

where $g_2 = 60 \cdot \sum_{\omega\in\Lambda,\omega\neq 0} \omega^{-4}$.