Knots and Numbers: Homework
18.095, January 6, 2020
Haynes Miller

1. The strands of a rational tangle, taken individually, are obviously unknotted. Prove the converse or suggest a counterexample.

2. Any rational number is of one of three forms: odd/even, even/odd, or odd/odd. What is the corresponding breakdown of rational tangles? How should ∞ be handled?

3. If K is a rational tangle with value q, what is the value of the mirror image of K?

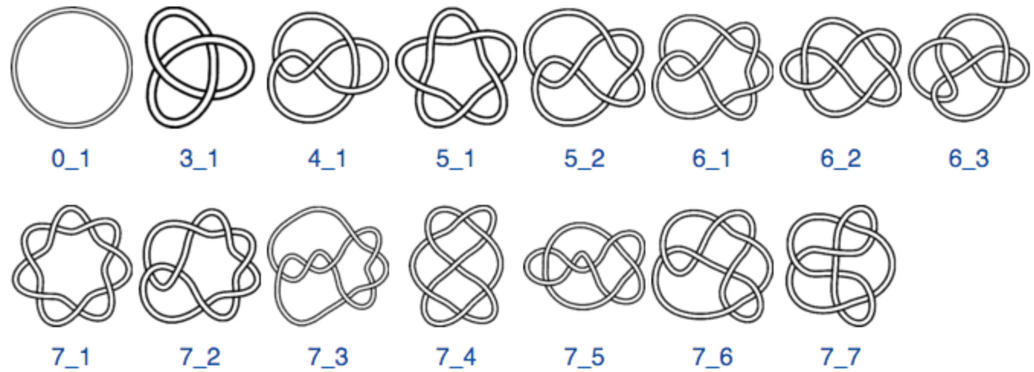
4. Prove that $R^2 = I$ on rational tangles. (This must be the case if the Schubert-Conway theorem is true, since $R^2 q = q$ for any rational number.) This operation rotates by 180° around an axis poking out of the page. How about 180° rotations around the other two principal axes - the x-axis and the y-axis? In fact, you may find it convenient to prove simultaneously that all three operations fix all rational tangles.

5. (For number theorists) Use the continued fraction of $q \in \mathbf{Q}$ to describe an algorithm for producing the rational tangle with value q. You may want to express the continued fraction using negatives:

$$a_0 - \cfrac{1}{a_1 - \frac{1}{a_2 - \cdots}}$$

6. A "two-bridge link" is a link that can be arranged in $\mathbf{R}^3$ in such a way that the z coordinate achieves just two local minima and two local maxima. Explain why such links are the same as links obtained by tying NE to NW and SE to SW in a rational tangle. Draw pictures of the link corresponding to the rational tangles with integral values. All prime knots with fewer than 8 crossings are 2-bridge knots. Can you draw pictures of them showing that? It turns out that exactly 12 of the 21 8-crossing prime knots are 2-bridge knots. I think it's rather hard to pick them out from the pictures!

Knots with 7 or fewer crossings



Knots with 8 crossings