In this pset, you will prove that the quantum state of a spin-1 particle that has spin 0 in the x-direction is orthogonal to the states with spin 0 in the y-direction and the z-direction, and thus that there is a measurement that projects the state onto one of these three states. This fact was used in my lecture when I was discussing the Kochen-Specker theorem.

Recall that the spin of a spin-$\frac{1}{2}$ particle can be represented as a superposition of spin up ($|\,0\rangle$) and spin down ($|\,1\rangle$). These states have spin $+\frac{1}{2}$ and $-\frac{1}{2}$ along the z-axis, respectively. See the figure for details — physicists call this figure the *Bloch sphere*.

If we have two spin-$\frac{1}{2}$ particles, then, as shown in the lecture, there is a state with spin 0 along any axis, namely

$$\frac{1}{\sqrt{2}}\Big(\,|\!\uparrow\rangle\otimes|\!\downarrow\rangle - |\!\downarrow\rangle\otimes|\!\uparrow\rangle\,\Big) = \frac{1}{\sqrt{2}}\Big(\,|\,01\rangle - |\,10\rangle\,\Big).$$

The 3-dimensional space of states orthogonal to this state is equivalent to the state space of a spin-1 particle.

The state of a spin-1 particle with 0 spin along the z axis must therefore be orthogonal to $|\,00\rangle$ (which has spin $+1$ along the z-axis) and $|\,11\rangle$ (which has spin -1 along the z-axis). And, since we're only considering the spin-1 subspace, it must also be orthogonal to $\frac{1}{\sqrt{2}}(|\,01\rangle - |\,10\rangle)$. Find this state.

Similarly, find the states of a spin-1 particle with 0 spin along the x-axis and with 0 spin along the y-axis. Show that these three states are orthogonal.