

Manhole Covers and Convex Geometry

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Without geometry life would be pointless.

Keywords

Shapes of constant width. Reuleaux triangle. Reuleaux tetrahedron. Minkowski sum. Meissner bodies.

Homework

Exercise 1. Among the shapes of constant width d , the circle has the largest area and the Reuleaux triangle has the smallest. Calculate both areas.

The circle of width d can hide behind the Reuleaux triangle of the same width, but it has a larger area. Calculate the area ratio.

Exercise 2. Calculate the largest width of the Reuleaux tetrahedron with side 1.

Exercise 3. Find the radius of largest circle that can hide behind a square of side 1. Can it hide inside? Find the largest square that can hide behind a circle of diameter 1. Can it hide inside?

Exercise 4. Find the largest sphere that can hide behind a regular tetrahedron with side 1. Can it hide inside?

Exercise 5. Calculate the area of the Minkowski sum of a disc of radius r and an equilateral triangle with side 1.

Open Problem. Optional

Exercise 6. Prove that the Meissner bodies have the smallest volume among the bodies of constant width.