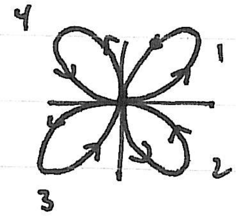
__Use__: lets you easily describe some curves that are more difficult to describe in $y=f(x)$ form. E.g.,

4-leaf rose $r = \sin 2\theta$



(Q: can give it parametric eqns?)

[Rmk: polar intersections are tricky b/c of multiple rep'ns. E.g., $r = 1+\cos^2\theta$ & $r = -1-\cos^2\theta$ are __same__ curve!]

Classical problems of calculus for polar curves:

- __Arclength__: $x = r\cos\theta \Rightarrow dx = dr\cdot\cos\theta - r\sin\theta\, d\theta$
  $y = r\sin\theta \Rightarrow dy = dr\cdot\sin\theta + r\cos\theta\, d\theta$

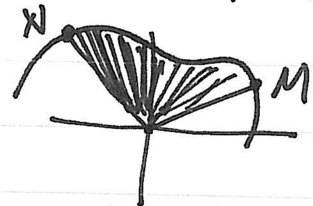
$$\Rightarrow ds^2 = dx^2 + dy^2 = (dr^2\cos^2\theta - 2r\cos\theta\sin\theta\, dr\, d\theta + r^2\sin^2\theta\, d\theta^2) + (dr^2\sin^2\theta + 2r\cos\theta\sin\theta\, d\theta + r^2\cos^2\theta\, d\theta^2)$$

$$= dr^2 + r^2\, d\theta^2$$

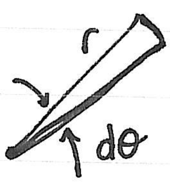
So, arclength of spiral $r = \theta^2$ for $0 \le \theta \le 2\pi$?

- __Tangent lines__: straightforward: use same idea, parametric eqns to find tangent vector.

Areas: Given a curve in polar "around the origin," how much area in the sector it defines?



Same idea as usual: cut into many little pieces, this time using a $d\theta$



$$dA = \frac{1}{2} r^2 d\theta$$

$$\Rightarrow \quad A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta$$

E.g., area inside one leaf of the 4-leaf rose $r = \sin 2\theta$?

$$\int_0^{\pi/2} \frac{1}{2} (\sin 2\theta)^2 d\theta$$

$$= \frac{1}{2} \int_0^{\pi/2} \left(\frac{1 - \cos 4\theta}{2} \right) d\theta$$

$$= \dots$$