Definite integrals & substitution

$$\int_0^2 v e^{v^2} dv$$

<u>Method 1</u>: $\int v e^{v^2} dv \overset{\boxed{\substack{u=v^2 \\ \text{etc}}}}{=} \frac{1}{2} e^{v^2}$

$$\Rightarrow \int_0^2 v e^{v^2} dv = \frac{1}{2}e^4 - \frac{1}{2}e^0 = \frac{e^4-1}{2}$$

<u>Method 2</u>: $\int_0^2 v e^{v^2} dv \underset{v=2 \to u=4}{\overset{\substack{u=v^2 \\ v=0 \to u=0}}{=}} \int_0^4 \frac{1}{2} e^u du$

$$= \frac{1}{2}e^4 - \frac{1}{2}e^0 = \frac{e^4-1}{2}.$$

Definite integration by parts

$$\int_a^b u\, dv = uv\Big]_a^b - \int_a^b v\, du$$

Differential equations

Often, one is in the position of wishing to compute a function given some indirect information about it, e.g., about its derivative. One simple example is antidifferentiation: given $\frac{dy}{dx} = f(x)$, we want to find $y = \int f(x)\,dx$. Often times we might face something like

$$\frac{dy}{dx} = -2xy^2 \Rightarrow \frac{dy}{y^2} = -2x\,dx$$

$$\xrightarrow{\text{integrate}} -\frac{1}{y} = -x^2 + c \quad \text{important!}$$

$$\Rightarrow y = \frac{1}{x^2+c}$$

(for some c).

Example application: a bank account that pays 1% interest compounded frequently (say, daily, so $\frac{1}{365}\%$ per day) roughly satisfies const, say 1%

$$\frac{dy}{dx} = c \cdot y$$

$$\rightsquigarrow \frac{dy}{y} = c\,dx$$

$$\Rightarrow \ln y = cx + C$$

$$\Rightarrow y = e^{cx+C} = De^{cx}$$

(where $D = y(0) =$ initial value)

This method is called <u>separation of variables</u>

& only works for the simplest diff eq'ns.