Learned idea of definite integration –

cutting a region up into little pieces

& relation to antiderivatives via Fund Thm:

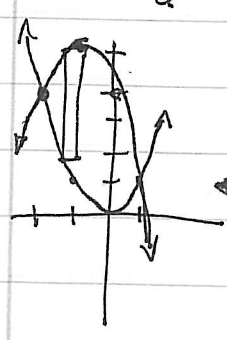
$$\int_a^b f(x)\,dx = F(b) - F(a)$$ where

F is an antiderivative of f, i.e.

$$\frac{dF}{dx} = f(x).$$

Examples of applications:

- computing areas. what is the area above the curve $y = x^2$ & below the curve $y = -x^2 - 2x + 4$?



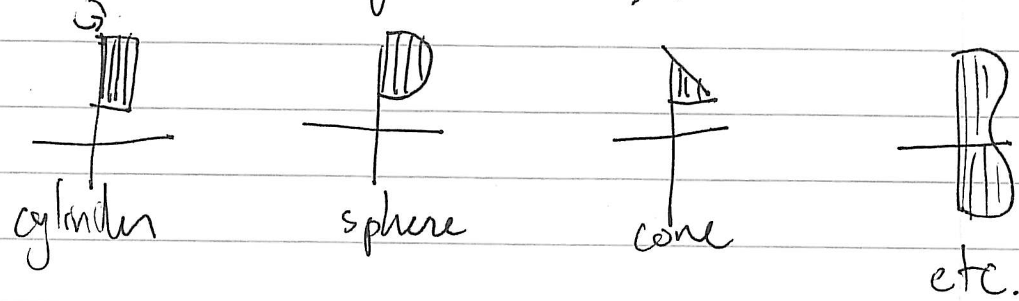
Sol'n:
Draw picture,

bounds: x from -2 to 1

$$\int_{-2}^{1} (-x^2 - 2x + 4) - x^2 \, dx$$

$$= \int_{-2}^{1} -2x^2 - 2x + 4 \, dx = -\frac{2x^3}{3} - x^2 + 4x \Big]_{-2}^{1} = 9$$

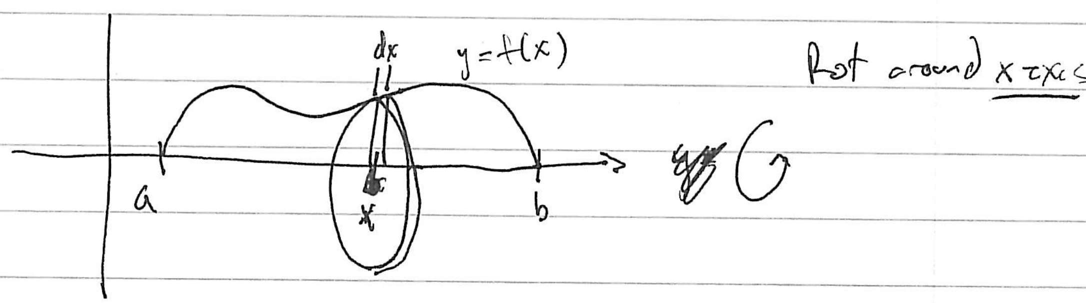
Second use: computing volumes, e.g., of certain solids of revolution, like



How to compute such a volume?

Method 1

Cut into a lot of disks



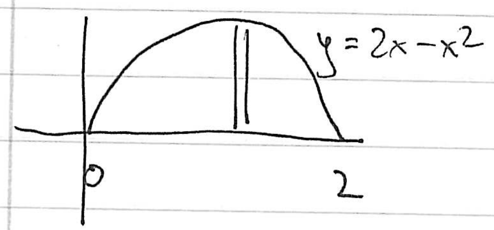
$$V_{disk} = A \cdot h = \pi r^2 \cdot h = \pi f(x)^2 dx$$

$$\Rightarrow \text{total volume} = \int_a^b \pi f(x)^2 \, dx$$

$$\left(= \int_a^b A(x) \, dx \right).$$

Ex 1: Suppose region between x-axis & $y=2x-x^2$ is rotated around the x-axis. Volume = ?

DRAW PICTURE!

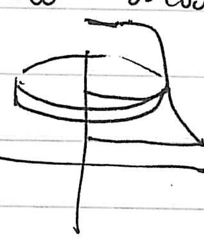


$$V_{slice} = \pi(2x-x^2)^2\,dx$$

$$V_{tot} = \int_0^2 \pi(2x-x^2)^2\,dx$$

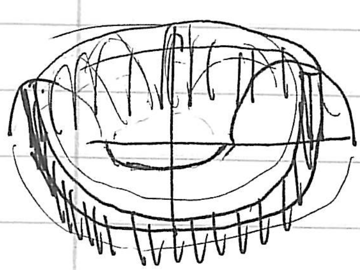
$$\left(= \pi \cdot \qquad \right)$$

Note: we can easily adjust this method to rotate around y-axis

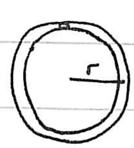


now integrate w.r.t. y

2nd Method: Cylindrical shells



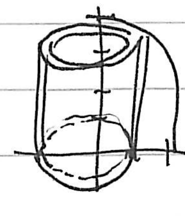
$$V_{shell} = \text{Area} \cdot \text{height} = 2\pi r\,dx \cdot f(x)$$



thickness = dx
area = $2\pi r\,dx$

(at least in this situation)

E.g. Region under $y = 4 - x^2$ in 1st quad around y-axis

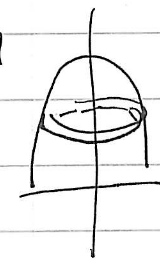


$$V_{shell} = 2\pi x(4-x^2)\,dx$$

$$V = \int_0^2 2\pi x(4-x^2)\,dx$$

$$=$$

Double-check w/ disks. $x = \sqrt{4-y}$



$$V_{disk} = \pi \sqrt{4-y}^{\,2}\,dy = \pi(4-y)\,dy$$

$$V = \int_0^4 \pi(4-y)\,dy$$

$$=$$

Shells ~~disks~~ are particularly good for solids w/ gaps.

Takeaway: $V_{disk} = \pi r^2 h$, $h = d$(variable of integration)

$$V_{shell} = 2\pi r h\,dr$$

Look at picture to choose r, h.

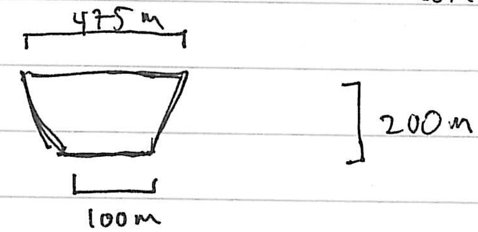
3rd method: Pappus' Theorem.

Other applications of integration: any time you want something similar, like hydrostatic force
(adding up little bits)

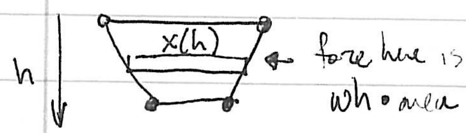
Fact: fluid pressure (= force/area) is given by $p = w \cdot h$ & is the same in all directions.

weight density — depth

Ex Dam:



$\Rightarrow$ total force on dam is



$$x(h) = 475 - \frac{375}{200}h$$

$$\int_0^{200} wh\left(475 - \frac{375}{200}h\right)\,dh$$

$$= w\left(\frac{475h^2}{2} - \frac{375}{600}h^2\right)\Big]_0^{200}$$

$$\approx 4{,}500{,}000\ w \cdot m^3$$

$$= 4.5 \times 10^9\ kg \cdot g$$

acceleration due to gravity