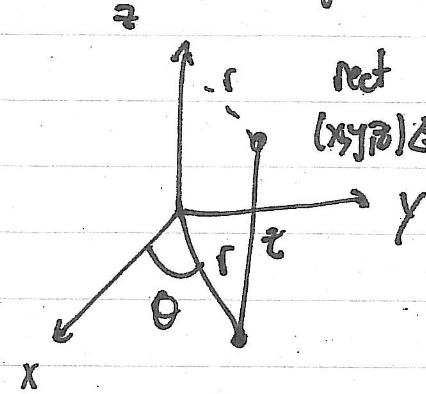
Cylindrical coordinates

Another useful coord. system: extends polar to 3D



rect $\quad$ polar

$(x,y,z) \Leftrightarrow (r,\theta,z)$

Translation:

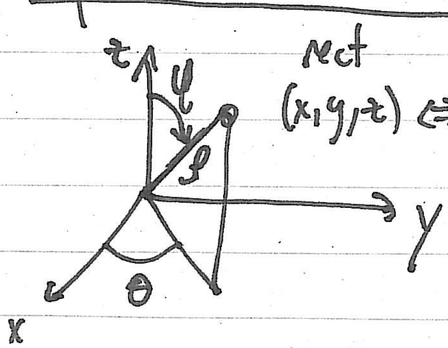
$$\begin{cases} x = r\cos\theta \quad y = r\sin\theta \quad z = z \\ r^2 = x^2+y^2 \quad \tan\theta = \dfrac{y}{x} \quad z = z \end{cases}$$

Simple graphs in cylindrical coords:

- $r = a$
- $\theta = \alpha$
- $z = a$
- $z = r$
- $z^2 + r^2 = a^2$

Cylindrical coordinates are most frequently useful when describing objects with a high degree of axial symmetry.

Spherical coordinates



rect $\quad$ spherical

$(x,y,z) \Leftrightarrow (\rho,\varphi,\theta)$

Translation

$x = \rho\sin\varphi\cos\theta \quad y = \rho\sin\varphi\sin\theta$

$z = \rho\cos\varphi$

$\rho^2 = x^2+y^2+z^2 \quad \tan\varphi = \dfrac{\sqrt{x^2+y^2}}{z} \quad \tan\theta = \dfrac{y}{x}$

Simple graphs in spherical coords

$\cdot\ \rho = a$ $\quad\cdot\ \theta = \alpha$ $\quad\cdot\ \varphi = \alpha$ where $\begin{cases} 0 \le \alpha < \frac{\pi}{2} \\ \alpha = \pi/2 \\ \pi/2 < \alpha \le \pi \end{cases}$

(Usually we'll think of $\rho \geq 0$, $0 \le \varphi \le \pi$.)

Quick check: how to translate spherical to cylindrical coordinates?

Triple integrals

Same idea: a 3D region cut into little boxes, to add up value of a function on that region

$$\iiint_R f(x,y,z)\, dV$$

$dV = dx\,dy\,dz$ $\longrightarrow$ in rectangular coordinates, 6 possible orders of integration when evaluating as an iterated integral

Same extensions of ideas from last time: if R is a solid w/ density $\delta(x,y,z)$ then

$$\text{Mass} = \iiint_R \delta \, dV \quad \& \quad \text{center of mass} = (\bar{x}, \bar{y}, \bar{z})$$

$$\text{w/} \quad \bar{x} = \frac{\iiint_R \delta \, dV}{\text{Mass}}, \quad \text{etc.}$$

When evaluating an triple integral as an iterated integral,

$$\iiint_R dV = \int_a^b \int_{y_1(x)}^{y_2(x)} \int_{z_1(x,y)}^{z_2(x,y)} dz\, dy\, dx$$

(note dependencies)

E.g. What is the center of mass of the tetrahedron w/ vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, $(0,0,3)$?
(Hint: slanted face has equation $6x+3y+2z=6$.)

E.g. $$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{1-x^2-y^2} f(x,y,z)\, dz\, dy\, dx = \int_?^? \int_?^? \int_?^? f(x,y,z)\, dx\, dy\, dz$$

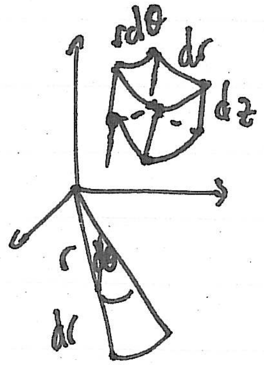
Hardest part: always the geometry of the situation
well, almost

One small observation: $\text{Vol}(R) = \iiint_R dV = \iiint_R 1\, dV$ & sim 17.4

Aside $\quad \text{Area}(D) = \iint_D dA = \iint_D 1\, dA$

What if we wanted to integrate in some other coordinate system?

Triple integrals in cylindrical coordinates: cut region into little pieces dr, $d\theta$, dz



$$dV = r\,dr\,d\theta\,dz$$
$$= r\,dz\,dr\,d\theta$$

E.g: Consider the solid inside the cylinder $x^2+y^2=1$, below the plane $z=2$ & above the paraboloid $z=1-x^2-y^2$. Density proportional to distance from z-axis. Mass $= ?$