$x = x(u,v), \quad y = y(u,v), \quad z = z(u,v).$

where (u,v) varies through some region D (the domain) in the u,v-plane. (as w/ parametric curves, each choice of parameters gives a point of the surface; set of all choices $\rightsquigarrow$ whole surface)

E.g. $\vec{r}(u,v) = \langle 2\cos u, \ v, \ 2\sin u \rangle$

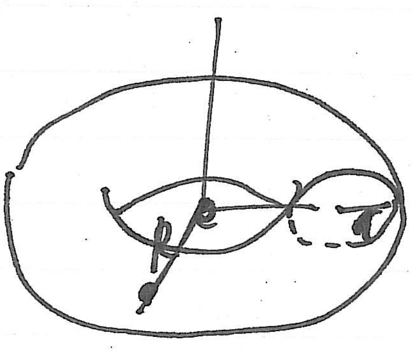
What's the surface?

$\boxed{16.1}$

E.g. $\vec{r}(u,v) = \langle x_0, y_0, z_0 \rangle + u\vec{a} + v\vec{b}$ where

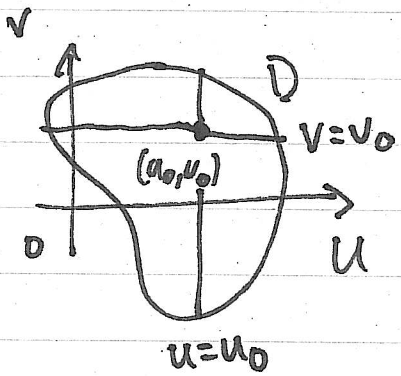
$\vec{a}, \vec{b}$ fixed 3D vectors (not parallel)

E.g. How to parametrize a torus?



Surface area of parametric surface

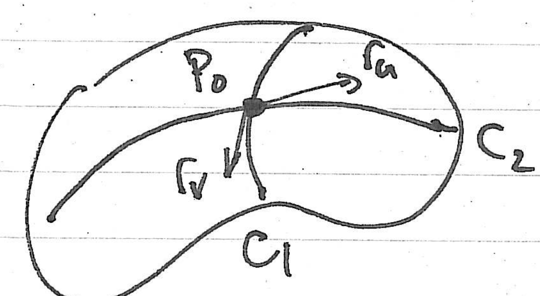
Idea: cut the surface into many little patches; approximate area of each patch by area of tangent plane.



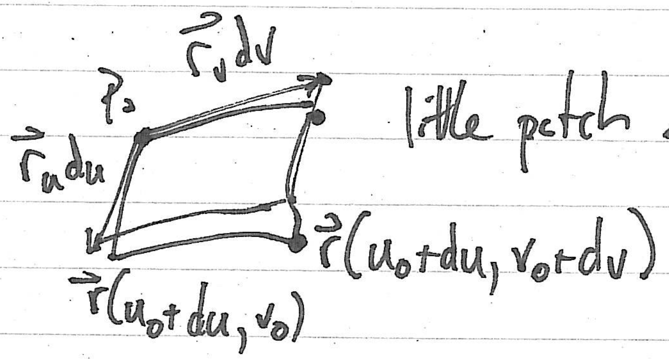
$P_0 = \vec{r}(u_0, v_0)$

$\vec{r}(u_0, v)$ is a curve on the surface (a "grid curve") w/ tangent vector

$\vec{r}_v(u_0, v_0) = \langle x_v(u_0,v_0), y_v(u_0,v_0), z_v(u_0, v_0)\rangle$



at the point P_0, & similarly w/ C_2, $\vec{r}(u, v_0)$, $\vec{r}_u(u_0, v_0)$.



little patch of surface associated to the ~~square~~ rectangle
(u_0, v_0) (u_0+du, v_0)
(u_0, v_0+dv) (u_0+du, v_0+dv)

in u,v-plane is closely approximated by the parallelogram w/ edges $\vec{r}_u\,du$ & $\vec{r}_v\,dv$

$$\Rightarrow \quad dSA = |\vec{r}_u du \times \vec{r}_v dv| = |\vec{r}_u \times \vec{r}_v|\,du\,dv$$

$$\Rightarrow \quad SA = \iint_D |\vec{r}_u \times \vec{r}_v|\,du\,dv$$

E.g. We saw that torus has parametric eqns

$$\vec{r}(u,v) = \langle (b+a\cos u)\cos v, (b+a\cos u)\sin v, a\sin u \rangle.$$

Thus $\quad \vec{r}_u = \langle -a\sin u\cos v, -a\sin u\sin v, a\cos u \rangle$

& $\quad \vec{r}_v = \langle -(b+a\cos u)\sin v, (b+a\cos u)\cos v, 0 \rangle$

$$\& \quad SA = \int_0^{2\pi}\int_0^{2\pi} \left| \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ - & - & - \\ - & - & - \end{vmatrix} \right| du\,dv$$

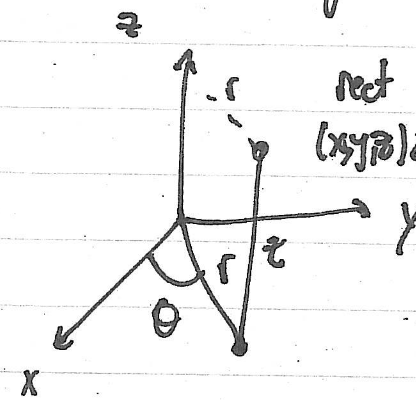
$$= \iint \quad \text{etc.}$$

HW: derive the formula for SA of a surface $z = f(x,y)$. (Maybe also for surfaces of rotation?)

16.4

Cylindrical coordinates

Another useful coord. system: extends polar to 3D



rect polar
$(x,y,z) \Leftrightarrow (r,\theta,z)$

Translation:

$$\begin{cases} x = r\cos\theta \quad y = r\sin\theta \quad z = z \\ r^2 = x^2+y^2 \quad \tan\theta = \frac{y}{x} \quad z = z \end{cases}$$

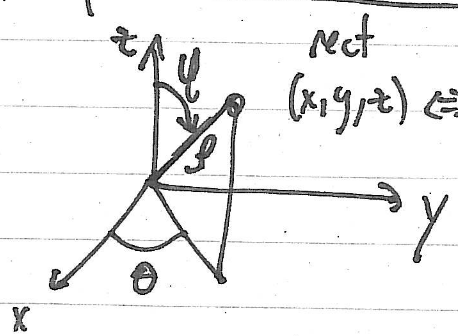
Simple graphs in cylindrical coords:

- $r = a$
- $\theta = \alpha$
- $z = a$
- $z = r$
- $z^2 + r^2 = a^2$

Cylindrical coordinates are most frequently useful when describing objects with a high degree of <u>axial</u> symmetry.

17.1

Spherical coordinates



rect spherical
$(x,y,z) \Leftrightarrow (\rho,\varphi,\theta)$

Translation

$x = \rho\sin\varphi\cos\theta \quad y = \rho\sin\varphi\sin\theta$

$z = \rho\cos\varphi$

$\rho^2 = x^2+y^2+z^2 \quad \tan\varphi = \frac{\sqrt{x^2+y^2}}{z} \quad \tan\theta = \frac{y}{x}$