Suppose we have a function $f(x_1, x_2, \dots, x_n)$ in several variables. When we move one of the x_i slightly, we measure change in f via the partial derivative $\frac{\partial f}{\partial x_i}$ (e.g., for $g(x,y)$, we have $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$; for $h(x,y,z)$, also $\frac{\partial h}{\partial z}$).

This is movement in one of the coord directions. What about instantaneous change if we move in some other direction?

$\hat{u} = \langle a, b \rangle$ unit vector; we want

$$D_{\hat{u}} f = \lim_{\Delta s \to 0} \frac{f(x_0 + a\Delta s,\ y_0 + b\Delta s) - f(x_0, y_0)}{\Delta s},$$

the <u>directional derivative</u> in the direction $\hat{u}$ @ point (x_0, y_0)

Ok, but how do we actually calculate it?

Well, know from yesterday that

$$f(x_0 + \Delta x,\ y_0 + \Delta y) - f(x_0, y_0) \approx f_x(x_0, y_0)\Delta x + f_y(x_0, y_0)\Delta y$$

so $f(x_0+u_1\Delta s,\ y_0+u_2\Delta s)-f(x_0,y_0) \overset{=\Delta f}{\approx}$

$f_x(x_0,y_0)\,u_1\Delta s + f_y(x_0,y_0)\,u_2\,\Delta s$ when Δs small,

so $$\frac{\Delta f}{\Delta s} \approx f_x(x_0,y_0)\,u_1 + f_y(x_0,y_0)\,u_2$$

$$= \langle f_x(x_0,y_0), f_y(x_0,y_0)\rangle \cdot \hat{u}$$

& in the limit this becomes the equality

$$D_{\hat{u}} f\,(x_0,y_0) = \langle f_x(x_0,y_0), f_y(x_0,y_0)\rangle \cdot \hat{u}$$

Similarly, if f is a function of 3 or more variables,

$$D_{\hat{u}} f\,(x,y,z) = \langle f_x(x,y,z), f_y(x,y,z), f_z(x,y,z)\rangle \cdot \hat{u}$$

(& sim. for fns of more vars)

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We denote $\langle f_x, f_y \rangle = \text{grad } f = \nabla f$

$$= \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath}$$

(+ same for 3-var or n-var)

(*) & so $D_{\hat{u}} f = \nabla f \cdot \hat{u}$

Since $\hat{u}$ is a unit vector, this says "the directional derivative of f in direction $\hat{u}$ is the magnitude of the projection of ∇f onto $\hat{u}$."

So ∇f summarizes all the info. on direct. derivatives that we could want to know.

E.g. If $f(x,y,z) = x^2 - y + z^2$

dir. deriv @ $(1,2,1)$ in dir. $\langle 4,-2,4 \rangle$

Max change? etc.