18.089 HW 5

1. For each of the following vector fields, check whether the field is conservative; if so, find the corresponding potential function.

a) $\vec{F} = \langle \cos x + ye^x, e^x - \sin y \rangle$

b) $\vec{F} = \langle 2x^2y + x, 2xy^2 + y \rangle$

c) $\vec{F} = \langle 2xy + 6xz, x^2 + 2z^2, 4yz + 2x^2 \rangle$

d) $\vec{F} = \langle yz + 1, xz + z - 1, xy + y + 2 \rangle$

2. Use the method described briefly at the end of class to find a potential function for $\vec{F} = \langle 3x^2 + z, 1, x \rangle$: let C be the straight line ~~curve~~ joining $(0,0,0)$ to (a,b,c) & define $f(a,b,c) = \int_C \vec{F} \cdot d\vec{R}$. (Compute the line integral by finding a simple parametrization

of C.) Verify that $\nabla f = \vec{F}$.

3. Evaluate $\oint_C (3x+4y)\,dx + (2x+3y^2)\,dy$

where C is the triangular path with vertices (2,0), (0,2) and (0,0). Do this by breaking the path into 3 parts, parametrizing & integrating each part separately.

4. Evaluate the integral of question 3 by converting it to a double integral using Green's Theorem.

5. We gave a formula for the area of a region R using Green's Theorem. Use this formula to compute the area bounded by the curve

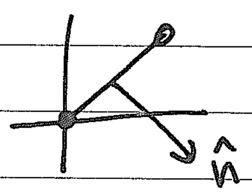
$$x = a\cos^3 t, \quad y = a\sin^3 t, \quad 0 \le t \le 2\pi$$

(Hints: $q^2r^4 + q^4r^2 = q^2r^2(q^2+r^2)$; $\sin t\cos t = \frac{1}{2}\sin(2t)$; $\sin^2\theta = (1-\cos(2\theta))/2$.)

6. Let $\vec{F}$ be the field $\langle 2x+2y, -4y \rangle$

a) How much work is done by $\vec{F}$ moving a particle along the straight line from $(0,0)$ to $(1,1)$?

b) ~~If $\vec{F}$ is a fluid flow~~, Thinking of $\vec{F}$ as a fluid flow, what is the flux of $\vec{F}$ across the segment joining $(0,0)$ to $(1,1)$?



(Use the normal $\hat{n}$ that points south-east.)