18.089 Homework 2

Summer 2010

Due Monday, June 14

1. Graph the curve defined by the parametric equations $y = \sin t$, $x = \sin t$ as t varies over all of $\mathbb{R}$. How would you describe this curve in rectangular coordinates (i.e., without using a parameter)?

2. At time t, a particle has position vector given by $\mathbf{R}(t) = \langle \cos t, \sin t, t \rangle$. Compute its velocity, acceleration and speed as functions of t.

3. Let ℓ be the line passing through the points $(0, 1)$ and $(1, 0)$ (in rectangular coordinates). Given three equations for this line: one in rectangular coordinates, one in parametric coordinates and one in polar coordinates.

4. What is the intersection of the line $\frac{x-1}{1} = \frac{y}{2} = \frac{z+2}{-2}$ and the plane $x + y + z = 3$?

5. Compute the arclength of a single arch of the cycloid generated by a circle of radius r.

6. Suppose a particle moves according to $\mathbf{R}(t) = e^t \cos t\, \hat{\mathbf{i}} + e^t \sin t\, \hat{\mathbf{j}}$. Compute its velocity and acceleration vectors. Then find its speed, and the magnitude of the tangential and normal components of acceleration.

7. The curve given in polar coordinates by $r = 1 - \sin\theta$ is called a *cardioid*.

   (a) Sketch the cardiod.

   (b) Compute its arclength.

   (c) Compute the area that it bounds.

   (You may want to use the half-angle formula $\sqrt{\frac{1-\cos\theta}{2}} = \sin\frac{\theta}{2}$ for $0 \leq \theta \leq 2\pi$. (For other values of θ, the two sides may disagree by a sign.))

8. The curve given in polar coordinates by $r = 1 + 2\sin\theta$ is called a *limaçon*. Sketch it – note in particular the existence of an inner and outer loop. Compute the area bounded by the inner loop.

9.