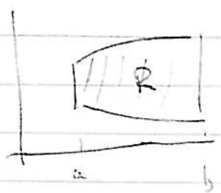
differentials! independent vars?

Multiple integrals / iterated integrals

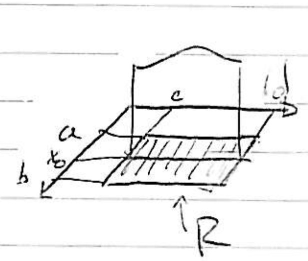
Volumes! (No notion of antiderivatives here.) Summing a function over some region. 2-var case: double integrals

Suppose



$f(x,y) \geq 0$ on R, want to know volume above R & below R. How to compute?

Look at slices, add them up! E.g., suppose (simplest case) that R is a rectangle

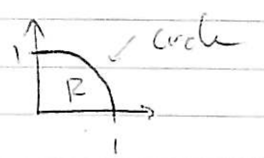


$$A_{slice} = \int_c^d f(x_0, y)\, dy$$

$$\Rightarrow V = \int_a^b A_{slice}\, dx = \int_a^b \int_c^d f(x,y)\, dy\, dx.$$

(Of course, this is same as $\int_c^d \int_a^b f(x,y)\,dx\,dy$.)

Could also have more complicated region, e.g.



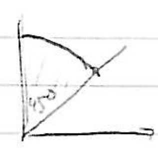
$$\text{Volume under } xy = \int_0^1 \int_0^{\sqrt{1-x^2}} xy \; dy\, dx$$

Evaluate inside first:

$$\left.\frac{xy^2}{2}\right|_0^{\sqrt{1-x^2}} = \frac{x}{2}(1-x^2)$$

$$V = \int_0^1 \frac{x}{2}(1-x^2)\, dx = \left.-\frac{1}{8}(1-x^2)^2\right|_0^1 = \frac{1}{8}$$

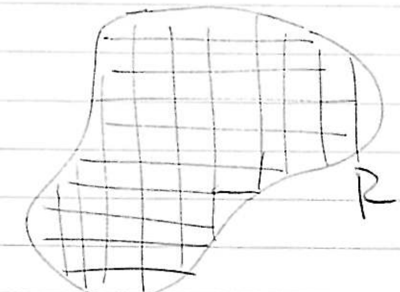
Q: what if we wanted volume over



?

Double integral: $\iint_R f(x,y)\,dA$: (for area)

idea is just like single integral: cut region up into lots of (approximate) rectangles, estimate region above each rectangle by a rectangular solid box, add up to get volume.



R

dA = "with respect to area" of the region we're integrating over, i.e., we're taking an infinitesimal element of area.

(& we could write this down as a limit of a sum, etc.)

One thing to note $\iint_R dA = A$.

(Comment: existence of double integrals has to do with properties of both R & f ... for our purposes, they'll always be ok as long as we avoid exploding values)

How are double integrals different from iterated integrals?

- Not very: an iterated integral is just a double integral plus an order of integration ("instructions on how to compute").
- But not every double integral can be converted into a single iterated integral: consider $\iint_R f(x,y)\,dA$ where

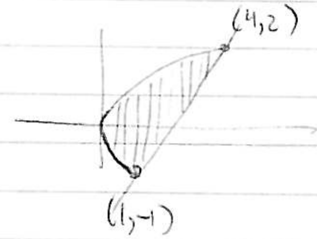
R = 

Consequence: always draw the region when playing w/ double or iterated integrals!

- Sometimes, which order you use matters:

For example, let R be the region bounded by $x=y^2$ & $x-y=2$ & compute $\iint_R 1+2x \; dA$

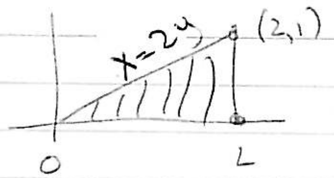
DRAW!



Which order is nicer?

(Hint: properties of the region.)

E.g. 2 $\quad R =$



$$\iint_R 4e^{x^2}\, dA$$

which order? (Hint: properties of the function.)

Well, what are these useful for?

- Have seen we can express volume by a double integral.
- Also area $\iint_R 1 \; dA = A$.
- Also mass: If $\delta = \delta(x,y)$ is the density of a thin plate then $\delta(x,y)\,dA$ is the element of mass of this plate & total mass is $m = \iint_R \delta(x,y)\; dA$.
- The center of mass is the point $(\bar{x}, \bar{y})$ given by

$$\bar{x} = \frac{\iint_R x\,\delta(x,y)\; dA}{m}$$

$$\bar{y} = \frac{\iint_R y\,\delta(x,y)\, dA}{m}.$$