

18.089 Exam 3

75 minutes

Tuesday, June 30, 2009

Name:

This exam consists of five problems, some of which have many parts, not arranged in any particular order. (Note that pages are printed front and back!) Please solve all problems in the space provided, showing all work as neatly and clearly as possible. **Be sure to read the problems carefully!**

You are allowed two (8.5 by 11 inch or equivalent) pages of notes for this exam. You may use a calculator, but please no sophisticated graphing or algebraic manipulation capabilities. All work must be your own.

Problem	Value	Score
Problem 1	15	
Problem 2	10	
Problem 3	22	
Problem 4	28	
Problem 5	15	
Problem 6	10	
Total	100	

Problem 1. (15 points) Consider the function $f(x, y) = \frac{\sin y}{x^2 + 1}$. This function has a unique critical point (x_0, y_0) with the property that $-1 \leq x_0 \leq 1$ and $0 \leq y_0 \leq 3$. Find this critical point and determine whether it is a maximum, minimum or saddle point.

Problem 2. (10 points) Write down a system of equations whose solutions give the points at which the function $f(x, y, z) = x^2 + 4yz$ reaches its extrema, subject to the constraint $x^2 + y^2 + z^2 = 1$. (You do not need to solve this system.)

Problem 3. ($4 + 4 + 4 + 4 + 2 + 4 = 22$ points) All parts of this problem refer to the surface with equation $z = xy - x$.

(a) Sketch four level curves of the surface, at height (z -value) -1 , 0 , 1 and 2 .

(b) Find and classify (maximum, minimum, saddle point) all critical points of the surface.

(c) Compute the gradient of the surface at the point $(1, 1, 0)$.

(d) Compute the tangent plane to the surface at the point $(1, 1, 0)$.

(e) Compute the directional derivative of the surface at the point $(1, 1, 0)$ in the direction $\langle -2, 3 \rangle$.

(f) Compute the volume of the solid bounded below the surface and above the triangular region R bounded by the lines $y = 2$, $x = 0$ and $y = \frac{x}{2} + 1$.

Problem 4. ($4 + 4 + 4 + 4 + 4 + 4 + 4 = 28$ points) All parts of this problem refer to the curve given by the parametric equations $x = t^2$, $y = t^3$, $z = t^5 - t$. Note that this curve passes through the point $(1, 1, 0)$ when $t = 1$.

(a) Find the velocity of the curve at the point $(1, 1, 0)$.

(b) Find the acceleration of the curve at the point $(1, 1, 0)$.

(c) Find the speed of the curve when at the point $(1, 1, 0)$.

(d) Find the unit tangent vector to the curve at the point $(1, 1, 0)$.

(e) Find the symmetric equations for the tangent line to the curve at the point $(1, 1, 0)$.

(f) The curve also passes through the point $(0, 0, 0)$. Write down an integral that gives the arclength of the curve between $(0, 0, 0)$ and $(1, 1, 0)$.

(g) Find all intersection points of the curve with the surface $z = xy - x$ of Problem 4.

Problem 5. (15 points) Let D be the solid consisting of points (x, y, z) that satisfy $y \geq 0$, $1 \leq x^2 + y^2 \leq 9$ and $-1 \leq z \leq 1$. (This solid looks like half of a cylindrical shell.) The density of D is given by $\delta(x, y, z) = x^2 + y^2 + z^2$. Compute the y -coordinate of the center of mass of D .

Problem 6. (10 points) A curve given by the polar equation $r = 2\sqrt{\cos 2\theta}$ for $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ is shown below. Compute the area it bounds.

