18.089 Exam 2

75 minutes

Wednesday, June 17, 2009

Name:

This exam consists of eight problems, not arranged in any particular order. (Note that pages are printed front and back, including the back of the last page!). Please solve all problems in the space provided, showing all work as neatly and clearly as possible. **Be sure to read the problems carefully!**

You are allowed one (8.5 by 11 inch or equivalent) page of notes for this exam. You may use a calculator, but please no sophisticated graphing or algebraic manipulation capabilities. All work must be your own.

Problem	**Value**	**Score**
Problem 1	11	
Problem 2	11	
Problem 3	15	
Problem 4	10	
Problem 5	10	
Problem 6	15	
Problem 7	12	
Problem 8	16	
Total	100	

Problem 1. *(5 + 6 = 11 points)* Let $M = \begin{pmatrix} a & 0 & b \\ 0 & 1 & 0 \\ -b & 0 & a \end{pmatrix}$.

(a) Compute $M \cdot M$.

(b) Compute M^{-1}.

Problem 2. *(5 + 6 = 11 points)* Let $\mathbf{U} = \langle 1, 1, 0 \rangle$, $\mathbf{V} = \langle 1, 0, 1 \rangle$.
(a) Compute the angle between $\mathbf{U}$ and $\mathbf{V}$.

(b) Compute $\mathbf{W} = \mathbf{U} \times \mathbf{V}$ and then compute $\mathbf{U} \cdot \mathbf{W}$.

Problem 3. *(10 + 5 = 15 points)* Define $I_n = \int x(\ln x)^n \, dx$.

(a) Use integration by parts to write I_n in terms of I_{n-1}.

(b) Compute I_3. (It might be helpful to use $I_0 = \int x \, dx$.)

Problem 4. *(10 points)* Compute the limit $\lim_{x \to 0^+} x(\ln x)^2$. (Note that this problem has nothing whatsoever to do with problem 3.)

Problem 5. *(10 points)* Does the integral

$$\int_1^\infty \frac{(x^2+1)\ln x}{x^3}\,dx$$

converge or diverge? (Explain!)

Problem 6. *(5 + 5 + 5 = 15 points)* Decide whether the following series converge or diverge. (Explain!)

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(n^2+1)\ln n}{n^3}$.

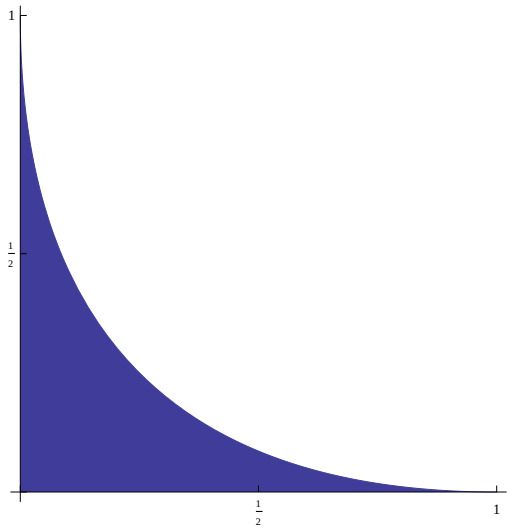
(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n(n^2+1)\ln n}{n^3}$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{3n}{n+1} \cdot \frac{1}{2^n}$.

Problem 7. *(12 points)* Consider the region in the first quadrant bounded by the curve

$$y = (1 - \sqrt{x})^2$$

for $0 \leq x \leq 1$, shown below. This region is rotated around the x-axis, forming a solid of rotation. Write down **two integrals** that give the volume of this solid, one using disks and the other using shells. *You do not need to evaluate these integrals!*



Problem 8. *(16 points)* Compute the first four nonzero terms (that is, the constant, x, x^2 and x^3 terms) of the power series for the function

$$f(x) = e^{x+x^2}$$

using any method you like.