18.089 Exam 1

75 minutes

Friday, June 5, 2009

Name:

This exam consists of seven problems, not arranged in any particular order. (Note that pages are printed front and back). Please solve all problems in the space provided, showing all work as neatly and clearly as possible.

This exam is open-note and open-text. You may use a calculator so long as it lacks sophisticated graphing capabilities. All work must be your own.

Problem	Value	Score
Problem 1	10	
Problem 2	15	
Problem 3	20	
Problem 4	10	
Problem 5	10	
Problem 6	15	
Problem 7	20	
Total	100	

Problem 1. *(10 points)* Compute the derivative $\frac{d}{dx} \ln(3^x + 1)$.

Problem 2. *(15 points)* Compute the arclength of the curve $y = \ln \cos x$ for $\frac{\pi}{6} \leq x \leq \frac{\pi}{4}$.

Problem 3. *(20 points)* Compute $\displaystyle\int_3^4 \frac{x^3 + 4x^2}{(x-1)(x^2 - 4x + 8)}\,dx.$

Problem 4. *(10 points)* What is the tangent line to the curve $y = 2 - e^{2-2x}$ at $x = 1$?

Problem 5. *(10 points)* What is the third derivative of $f(x) = \sin x \cos x$?

Problem 6. *(15 points)* A special balloon has the property that no matter its volume, it always maintains a perfectly cubical shape. The balloon is being inflated at a rate of 6 cubic centimeters per second. When the balloon has volume 64 cubic centimeters, how fast is its surface area increasing?

Problem 7. *(20 points)* Compute the values of a and b so that the function f defined by

$$f(x) = \begin{cases} x^3, & x \geq 1 \\ x^2 + ax + b, & x < 1 \end{cases}$$

is differentiable for all values of x. For this value of a and b, sketch the curve defined by $y = f'(x)$.