MIT Dept. of Mathematics
Benjamin Seibold

18.086 spring 2007

Exercise Sheet 4

Out Mon 04/02/07
Due Wed 04/18/07

We would like to solve a Poisson problem on the 2d geometry given in Figure 1, and the 3d geometry given in Figures 2 and 3.

Let the interior of the domains be denoted Ω, and the boundary denoted Γ. In the 2d geometry, the boundary consists of lines, in the 3d geometry, the boundary is composed of faces. The Poisson problem to be solved is

$$\begin{cases} -\Delta u = 1 & \text{in } \Omega \\ u = f & \text{on } \Gamma_D \\ \frac{\partial u}{\partial n} = \frac{\partial f}{\partial n} & \text{on } \Gamma_N \end{cases}$$

Let the function f, which defines the boundary conditions, be

- $f(x, y) = x^3y - xy^3 - \frac{1}{2}x^2$ in the 2d case,
- $f(x, y, z) = x^3yz + xy^3z - 2xyz^3 - \frac{1}{2}x^2$ in the 3d case.

Let Dirichlet boundary conditions be specified where boundary lines are thin and faces are light, and Neumann boundary conditions be specified where boundary lines are thick and faces are dark.

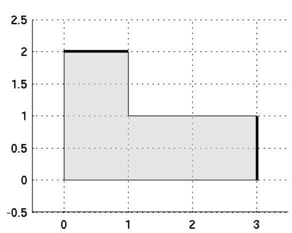

Fig 1: Tetris geometry (2d)

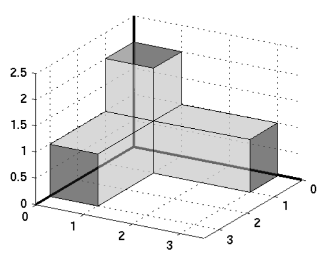

Fig 2: Blockout geometry (3d) from the front

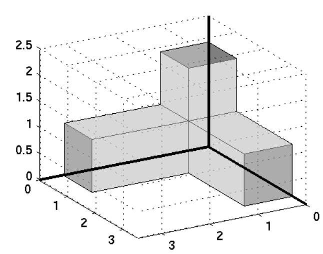

Fig 3: Blockout geometry (3d) from behind

Exercise 8 Write a matlab program which takes a grid resolution parameter as input and sets up a linear system discretizing the above 2d and 3d Poisson problems.

Exercise 9 For both the 2d and the 3d problem, choose a small resolution, a medium resolution and a high resolution, and (try to) solve the linear systems by

1. elimination with reordering
2. Jacobi iteration
3. a simple multigrid method (V-cycle)
4. conjugate gradients

Compare the different approaches with respect to accuracy and run times.

Remarks:

- For elimination, matlab functions can be used. The other three methods have to be implemented by hand (in a simple version).
- In all cases, the numerical error due to solving the linear system shall not be significantly larger than the error due to the discretization of the geometry.
- Some solution methods might fail for very large resolutions. The medium resolution shall be chosen, such that all above solution methods succeed.
- The high resolution shall drive matlab to its limits. Fame and glory for the student who can manage the largest resolution in his/her homework.