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Exercise Sheet 2

Out Wed 02/21/07
Due Fri 03/09/07

Exercise 2 Look at the matlab file `mit18086_error_analysis.m` on the course web page.

1. Run the program with the three different initial conditions provided in the file. Explain the results.
2. Modify the Lax-Wendroff method by multiplying the added viscosity by a factor of 1.01. Compare with the original Lax-Wendroff scheme and explain the result.
3. Include the Crank-Nicolson method for the transport equation, as given in the file `mit18086_fd_transport_growth.m` and compare to the other methods.

Exercise 3 Consider the convection-diffusion equation

$$u_t = u_x + d u_{xx}$$

Implement the three candidate methods given in section 6.6 in the lecture notes, i.e.

1. forward in time, centered convection, centered diffusion
2. forward in time, upwind convection, centered diffusion
3. explicit convection, implicit diffusion

and test them, firstly for a large, secondly for a medium, and thirdly for a very small diffusion d. Explain your results.

Exercise 4 Consider Burgers' equation

$$u_t + u u_x = 0$$

with a Gaußian initial condition $u_0(x) = \exp(-x^2)$. Choose your computational interval large enough, such that the solution is essentially 0 at the boundaries.

Compare the numerical solution obtained by a nonconservative upwind method to the solution obtained by the conservative upwind method

1. up to a time at which the analytical solution is still smooth
2. up to about twice the time after a shock has appeared in the analytical solution.