Problem Set 1

Unless otherwise specified, you may use MATLAB to assist with computations. Please provide a print-out of the code used and its output with your assignment.

1. Find the dimensions of the four fundamental subspaces of the following matrices. Also, determine if the matrices are full-column rank, full-row rank, both, or neither. Explain your reasoning.

   (a) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

   (b) $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$.

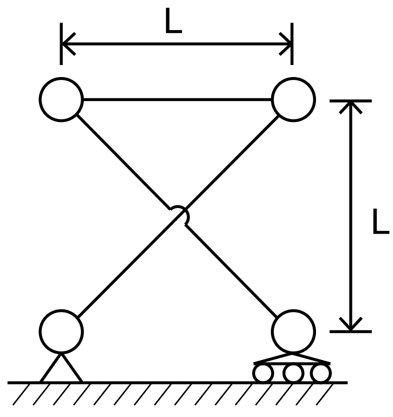
   (c) $vv^\top$, where v is a non-zero $n \times 1$ vector.

   (d) $u^\top u$, where u is a non-zero $n \times 1$ vector.

   (e) $A^\top A$, where A is a full-column rank $m \times n$ matrix.

   (f) $BB^\top$, where B is a full-row rank $m \times n$ matrix.

2. Consider the following truss. It is fixed to the ground on the left by a pin joint (allowing for rotation but no translation), and on the right by a roller (allowing for horizontal translation but no vertical translation).



   (a) Write down the 3×5 linear system, $Ax = 0$, that enforces the constraint that all bars are of fixed length.

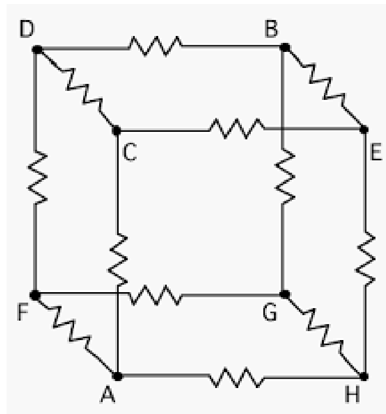
   (b) What is the number (p) of independent modes of deformation?

   (c) Find p independent modes of deformation and sketch them.

   (d) What is the minimum number of additional bars required to stabilize this truss?

   (e) Adding only this minimum number of bars, design a stable truss (without changing the supports). Demonstrate that it is stable (show that there are no deformation modes that preserve bar length).

3. Find the equivalent resistance of the following cube of resistors (each individual resistor has resistance R) between the points A and B.



4. **This problem is to be done by hand.** How many solutions are there to the following linear systems? Find *all* solutions, if any. If the problem has a unique solution, solve by Gaussian elimination on the augmented matrix.

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$