18.085: EXAM 2

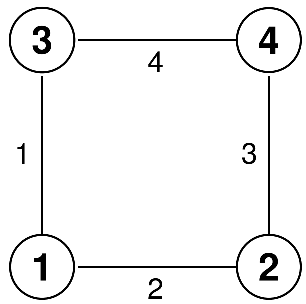
July 28, 2014

NAME:

SCORE:

Question 1. (25 pts.)

Consider the following plane square truss:



(a) (5 pts.) Write down the matrix A for this truss.

(b) (3 pts.) What is the rank of A?

Hint: It is probably easiest to determine the number of independent rows.

(c) (5 pts.) How many instabilities (rigid motions and mechanisms) does this truss have? Write down the corresponding displacement vector(s) $\vec{u}$.

Hint: Recall from earlier in the course that

$$\text{rank}(A) + (\text{number of independent solutions to } A\vec{u} = \vec{0}) = \text{number of columns in } A$$

(d) (3 pts.) Now fix nodes 1 and 2. Write down the matrix A_1 corresponding to this new truss.

(e) (4 pts.) What is the rank of A_1, and how many instabilities does the truss have? Write down the corresponding displacement vector(s) $\vec{u}$.

(f) (5 pts.) For the truss in part (d) (with nodes 1 and 2 fixed), add a bar between nodes 1 and 4. Write down the new matrix A_2. How many mechanisms does this truss have? Explain your answer mathematically; do not give purely physical arguments!

Question 2. (10 pts.)

Consider the equation

$$x^3 + 4 = 2x$$

Use Newton's method to find an approximate solution to this equation. Specifically, carry out two iterations of Newton's method starting with the initial guess $x_0 = 0$.

Question 3. (25 pts.)

We consider a non-uniform beam of length 2 hanging under the force of gravity. Specifically, we will solve the equation

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(c(x)\frac{\mathrm{d}^2u}{\mathrm{d}x^2}\right) = f(x), \quad 0 \le x \le 2$$

The force is $f(x) = -1$, and the stiffness function $c(x)$ is

$$c(x) = \begin{cases} 1 & \text{if } x \le 1 \\ x & \text{if } x > 1 \end{cases}$$

We assume the beam to be simply supported, so we take the boundary conditions

$$u = 0, \quad M = 0$$

at both ends, where $M(x) = c(x)\frac{\mathrm{d}^2u}{\mathrm{d}x^2}$ is the bending moment.

(a) (10 pts.) Find the bending moment $M(x) = c(x)\frac{\mathrm{d}^2u}{\mathrm{d}x^2}$. (Don't forget to impose the boundary conditions on M.)

(b) (5 pts.) Find u''.

(c) (10 pts.) Find the displacement $u(x)$ by imposing the boundary conditions on $u(x)$. Also impose the requirement that u and u' must be continuous along the beam. Once you impose these conditions, you should obtain equations that determine the constants in $u(x)$. **You do not have to solve the equations.**

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Question 4. (15 pts.)

Solve Laplace's equation $\Delta u = 0$ on the unit disc with the boundary condition

$$\frac{\partial u}{\partial r} = \cos 2\theta - \sin 4\theta$$

on the boundary of the disc.

Question 5. (25 pts.) In this question, we will set up the finite element method for the equation

$$-\frac{\mathrm{d}}{\mathrm{d}x}\left(c(x)\frac{\mathrm{d}u}{\mathrm{d}x}\right) = \delta(x-2/3), \quad 0 \le x \le 1$$

where

$$c(x) = \begin{cases} 2 & \text{if } x < 1/3 \\ 4 & \text{if } x > 1/3 \end{cases}$$

We take the boundary conditions

$$w(0) = u(1) = 0$$

where $w(x) = c(x)\frac{\mathrm{d}u}{\mathrm{d}x}$.

(a) (5 pts.) Write down the weak form of the differential equation. What condition must the test functions $v(x)$ satisfy?

(b) (5 pts.) Take $h = 1/3$. Draw the hat functions you will use to solve this problem.

(c) (10 pts.) Construct the matrix K for this problem. Show your work.

(d) (5 pts.) Construct the vector $\vec{F}$ for this problem. Show your work.

(e) **Bonus (5 pts.)**: Solve the equation $K\vec{U} = \vec{F}$ for $\vec{U}$ and write down the finite element solution $U(x)$. Graph the solution.