18.085 Quiz 3 December 9, 2005 Professor Strang

Your PRINTED name is: _____

Grading
1
2
3

1) **(30 pts.)** (a) Solve this *cyclic convolution* equation for the vector d. (I would transform convolution to multiplication.) Notice that $c = (5,0,0,0) - (1,1,1,1)$. The equation is like deconvolution.

$$c \circledast d = (4,-1,-1,-1) \circledast (d_0, d_1, d_2, d_3) = (1,0,0,0)\,.$$

(b) Why is there no solution d if I change c to $C = (3,-1,-1,-1)$? Try it. Can you find a nonzero D so that $C \circledast D = (0,0,0,0)$?

x

2) **(36 pts.)** (a) If $f(x) = e^{-x}$ for $0 \le x \le 2\pi$, extended periodically, find its (complex) Fourier coefficients c_k.

(b) What is the decay rate of those c_k and how could you see the decay rate from the function $f(x)$?

(c) Compute $\sum_{-\infty}^{\infty} |c_k|^2$ for those c's as an ordinary number. [1 point question: How in the world could you find $\sum_{-\infty}^{\infty} |c_k|^4$? Don't try!]

(d) Solve this periodic differential equation to find $u(x)$:

$$u'(x) + u(x) = \delta(x) + \delta(x + 2\pi) + \delta(x - 2\pi) + \cdots \text{ train of deltas}$$

xx

3) **(34 pts.)** Suppose $f(x)$ is a *half-hat function* $(-\infty < x < \infty)$.

$$f(x) = \begin{cases} 1 - x & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other } x \end{cases}$$

(a) Draw a graph of $f(x)$ on the whole line $-\infty < x < \infty$ and ALSO a graph of its derivative $g(x) = df/dx$.

(b) What is the transform (Fourier integral) $\widehat{g}(k)$ of df/dx?

(c) What is the transform $\widehat{f}(k)$ of $f(x)$? Does it have the decay rate you expect? What is $\widehat{f}(0)$?

(d) Christmas present: Is the convolution $f(x) * f(x)$ of the half-hat with itself equal to the usual full hat $H(x)$? (*Yes or no answer*, 4 points).

THANK YOU FOR TAKING 18.085! 18.086 will be good small projects in scientific computing.

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