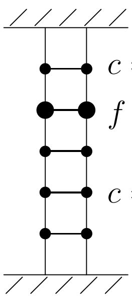
18.085 Quiz 2 November 14, 2005 Professor Strang

Your PRINTED name is: ______________________ **Grading** **1** **2** **3**

1) **(34 pts.)** A point load at $x = \frac{1}{3}$ hangs at the same point where $c(x)$ changes from $c = 1$ (for $0 < x < \frac{1}{3}$) to $c = 2$ (for $\frac{1}{3} < x < 1$). Both ends are FIXED.



(a) Solve for $u(x)$ and $w(x) = c(x)\,u'(x)$:

$$-\frac{d}{dx}\left(c(x)\,\frac{du}{dx}\right) = \delta\left(x - \frac{1}{3}\right) \quad \text{with} \quad u(0) = u(1) = 0\,.$$

(b) Draw the graphs of $u(x)$ and $w(x)$.

(c) Divide the hanging bar into intervals of length $h = \frac{1}{6}$ (then $c(x)$ changes from 1 to 2 at $x = 2h$). There are unknowns $U = (u_1, \ldots, u_5)$ at the meshpoints. Write down a matrix approximation $\boldsymbol{KU} = \boldsymbol{F}$ to the equation above. Take differences of differences (each difference over an interval of length h).

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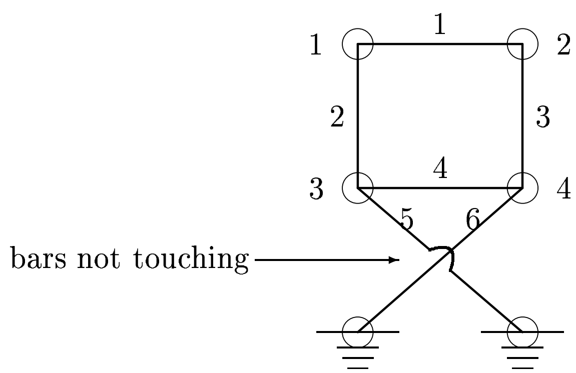
2) **(33 pts.)** This truss doesn't look safe to me. Those angles are 45°. The matrix A will be 6 by 8 when the displacements are fixed to zero at the bottom.



(a) How many independent solutions to $e = Au = 0$? Draw these mechanisms.

(b) Write numerical vectors $u = (u_1^{\mathrm{H}}, u_1^{\mathrm{V}}, \ldots, u_4^{\mathrm{H}}, u_4^{\mathrm{V}})$ that solve $Au = 0$ to give those mechanisms in part (a).

(c) What is the first row of $A^{\mathrm{T}}A$ (asking about $A^{\mathrm{T}}A$!) if unknowns are taken in that usual order used in part (b)?

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3) **(33 pts.)**

(a) Is the vector field $w(x, y) = (x^2 - y^2, 2xy)$ equal to the gradient of any function $u(x)$? What is the divergence of w? If $u(x, y)$ and $s(x, y)$ are a Cauchy-Riemann pair, show that $w(x, y) = (s(x, y), u(x, y))$ will be a gradient field and also have divergence zero.

(b) Take real and imaginary parts of $f(x + iy) = (x + iy + \frac{1}{x+iy})$ to find two solutions of Laplace's equation. Write those two solutions also in polar coordinates.

(c) Integrate each of the functions $u = 1, u = r\cos\theta, u = r^2 \cos 2\theta$ around the closed circle of radius 1 to find $\int u \, d\theta$. How could this same computation come from the Divergence Theorem?

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