

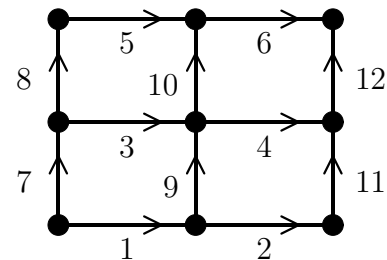
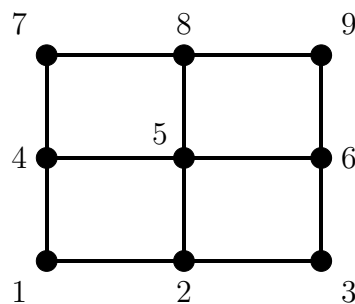
Your PRINTED name is: _____

Grading 1

2

3

- 1) (40 pts.) Here is a network with 9 numbered nodes and then separately the 12 edges.



- What is the shape of the incidence matrix A ? What is its 4th row?
- What is the 5th column of A ? What is the $(5, 5)$ entry in $A^T A$? Write the whole 5th row of $A^T A$.
- How many independent solutions to Kirchhoff's Law $A^T w = 0$? Find one of them.
- Without writing down this matrix A , explain why $A^T A$ is or is not symmetric positive definite.

- 2) (30 pts.)
- (a) For a hanging elastic bar, with $u(0) = 0$ at the top and $u'(0) = 0$ at the bottom and elastic constant $c(x) = 1$, what is the displacement $u(x)$ when a *unit point load* $f(x) = \delta(x - a)$ acts at the point $x = a$? Draw a graph of $u(x)$.
 - (b) What is the limit of $u(x)$ as the unit load moves to the bottom ($a \rightarrow 1$)? Suppose it moves to the top ($a \rightarrow 0$)? Draw graphs of $u(x)$ in those two cases.
 - (c) Choose a matrix equation that approximates the differential equation in part (a). (Describe the matrix—OK to put the load at a meshpoint.) If the load moves to the lowest meshpoint (number N), what displacements correspond to your answer in part (b)?

xx

3) (30 pts.) Suppose you measure your initial position $u_1 = b_1$, and then you measure the step lengths $u_2 - u_1 = b_2$ and $u_3 - u_2 = b_3$. At the end you make a last measurement $u_3 = b_4$.

- (a) Under what conditions on b_1, b_2, b_3, b_4 will these four equations have an exact solution? Create (don't solve) a set of equations for the best estimates $\hat{u}_1, \hat{u}_2, \hat{u}_3$.
- (b) Draw a picture of masses, springs, and forces (write in all constants) that would lead to the same equations for the displacements.
- (c) Suppose the variances for errors in the measurements are $\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2$. What equations should you solve (DON'T DO IT) for the statistically best estimate \hat{u} ? If $\sigma_4 \rightarrow \infty$ so that b_4 becomes completely unreliable, what answer do you expect for the best \hat{u} ?

xxx