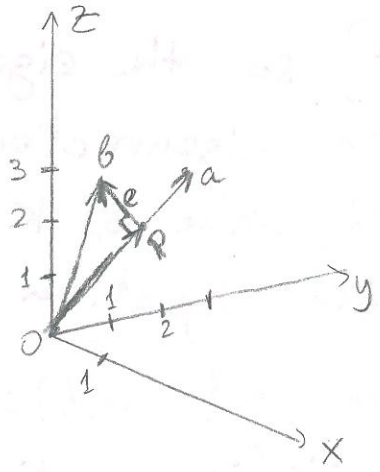
2. Projection onto a line in 3-dimensional space. The line contains all multiples ax of the vector $a = (1, 2, 3)$. We are given any vector $b = (b_1, b_2, b_3)$. We want to find the number $\widehat{x}$ so that $p = a\widehat{x}$ is the closest point to b, on the line through a.

(a) Draw a picture of a and b in $x - y - z$ space. Mark the closest point p, and mark the error $e = b - p = b - a\widehat{x}$. What equation will give the number $\widehat{x}$? FIND a formula for $\widehat{x}$.



Equation:

Want to solve $A\hat{x} = B$, where $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

$\hat{x}$ - 1×1 vector.

Least squares normal equation

$A^T A\hat{x} = A^T b$, i.e.

$14\hat{x} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = b_1 + 2b_2 + 3b_3$

Get $\hat{x} = \dfrac{b_1 + 2b_2 + 3b_3}{14}$

(Alternatively, use scalar product in 3-dim. space to get $p = a\hat{x}$ *, $\hat{x} = \dfrac{\langle a, b \rangle}{\|a\|^2} = \dfrac{b_1 + 2b_2 + 3b_3}{14}$).