18.085 Prof. Strang | Quiz 2 | November 7, 2003

Your name is: ______________________________ **Grading** 1
2
3

Total

1) **(36 pts.)** (a) For $-\frac{d^2u}{dx^2} = \delta(x-a)$ with $u(0) = u(1) = 0$, the solution is linear on both sides of $x = a$ (graph = triangle from two straight lines). *What is wrong with the next sentence*? Integrating both sides of the equation from $x = 0$ to $x = 1$, the area under the triangle graph is 1. The base is 1 so the height must be $u_{\max} = 2$.

(b) If $u(x, a)$ is the **true** solution to that standard problem in part (a), and the load point $x = a$ approaches $x = 1$, *what function does the solution u approach*? Give a physical reason for your answer or a math reason or both.

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2) **(40 pts.)** Which of these 5 equations can be solved?? If the equation has a solution, please find *one*. If not *why not*??

(a)

$$\operatorname{div} \begin{bmatrix} \partial u/\partial x \\ \partial u/\partial y \end{bmatrix} = 1$$

(b)

$$\operatorname{div} \begin{bmatrix} \partial s/\partial y \\ -\partial s/\partial x \end{bmatrix} = 1$$

(c) $u_{xx}+u_{yy} = 0$ in the unit circle and $u(1,\theta) = \sin 4\theta$ around the boundary

(d) Find a family of curves $u(x,y) = C$ that is everywhere perpendicular to the family of curves $x + x^2 - y^2 = C$.

(e) $d^4u/dx^4 = \delta(x)$ [point load at $x = 0$, not requiring boundary conditions, any solution $u(x)$ is OK].

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3) **(24 pts.)** I want to solve Laplace's equation (really Poisson's equation) in 3D with *no boundaries* (the whole space). The right side is a point load δ at the origin $(0,0,0)$. So $-\mathrm{div}(\mathrm{grad}\ u) = \delta$.

(a) Integrate both sides over a sphere of radius R around the origin:

$$\iiint -\mathrm{div}(\mathrm{grad}\ u)\, dxdydz = 1\,.$$

Use the divergence theorem (or Gauss-Green) to transform that triple integral into an integral on the surface of the sphere of radius R.

(b) What is the normal vector n in that integral? Knowing that u must be radially symmetric, $\frac{\partial u}{\partial r}$ is a constant on the sphere of radius R. What is that constant?

(c) So what is $u(r)$?

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