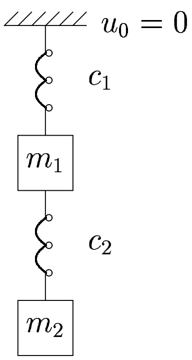
18.085 Quiz 1 Professor Strang October 6, 2003

Your name is: ____________________

Grading 1
2
3

Total

1) **(30 pts.)** A system with 2 springs and masses is **fixed-free**. Constants are c_1, c_2.



(a) Write down the matrices A and $K = A^{\mathrm{T}}CA$.

(b) Prove by **two tests** (pivots, determinants, independence of columns of A) that this matrix K is (positive definite) (positive semidefinite). **Tell me which two tests you are using**!

(c) Multiply column times row to compute the "element matrices" K_1, K_2:

Compute $K_1 =$ (column 1 of A^{T})(c_1)(row 1 of A)

Compute $K_2 =$ (column 2 of A^{T})(c_2)(row 2 of A) .

Then $K = K_1 + K_2$. What vectors solve $K_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$?

For those displacements x_1 and x_2, **what is the energy in spring 2**?

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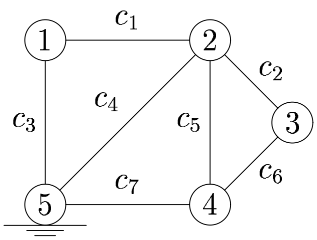
2) **(33 pts.)** A network of nodes and edges and their conductances $c_i > 0$ is drawn without arrows. Arrows don't affect the answers to this problem; the edge numbers are with the c's. Node 5 is grounded (potential $u_5 = 0$).



(a) List all positions (i, j) of the 4 by 4 matrix $K = A^{\mathrm{T}}CA$ that will have **zero entries**. What is row 1 of K?

(b) Find as many independent solutions as possible to Kirchhoff's Law $A^{\mathrm{T}}y = 0$.

(c) Is $A^{\mathrm{T}}A$ always positive definite for every matrix A? If there is a test on A, what is it? What is the trick that proves $u^{\mathrm{T}}Ku \geq 0$ for every vector u?

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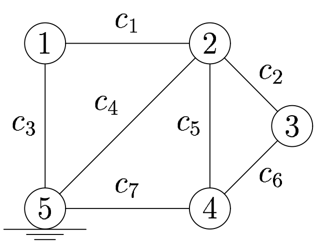
3) **(37 pts.)** Make the network in Problem 2 into a 7-bar truss! The grounded node 5 is now a supported (but turnable) pin joint, with known displacements $u_5^{\mathrm{H}} = u_5^{\mathrm{V}} = 0$. **All angles are** $45°$ **or** $90°$.



(a) How many rows and columns in the (reduced) matrix A, after we know $u_5^{\mathrm{H}} = u_5^{\mathrm{V}} = 0$? Describe in words (or a picture) all solutions to $Au = 0$. If you add 1 bar can A become square and invertible?

(b) Write out row 2 of A, corresponding to bar 2. Then (row 2) times the column u of displacements has what physical meaning?

(c) What is the first equation of $A^{\mathrm{T}}w = f$ (with right side f_1^{H})? Why does $\frac{1}{2}u^{\mathrm{T}}Ku = \frac{1}{2}y^{\mathrm{T}}C^{-1}y$ and what does this quantity represent physically? (More than 1 word in that last answer, less than 10 words.)

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