18.085 Profs Stroock, Stefanica and Strang Quiz 1 October 5, 2001

Your name is: ______________________________

Grading	1
	2
	3
Total	

1) **(24 pts.)** The key equation of least squares is $A^{\mathrm{T}}A\bar{x} = A^{\mathrm{T}}b$. The vector $\bar{x}$ solves the equation $Ax = b$ "as well as possible". The error vector is $e = b - A\bar{x}$.

(a) What does "as well as possible" mean?

(b) Write $A^{\mathrm{T}}A\bar{x} = A^{\mathrm{T}}b$ as $A^{\mathrm{T}}e = 0$. The first component of $A^{\mathrm{T}}e = 0$ says that some vector (which vector???) is orthogonal to e. From all n of the components of $A^{\mathrm{T}}e = 0$ we learn that e is orthogonal to ____________________ . Draw a picture and label e and b and $A\bar{x}$.

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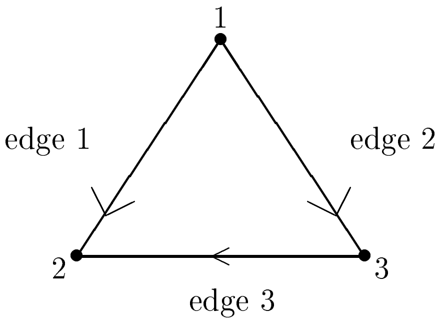
2) **(48 pts.)** (a) Write down the 3 by 3 incidence matrix A_0 for this simple triangle graph, and then write down the matrix A after node 3 has been grounded. Compute $K_0 = A_0^{\mathrm{T}} A_0$ and $K = A^{\mathrm{T}} A$.



(b) Suppose batteries $b_1 = 1$, $b_2 = 1$, $b_3 = 1$ are inserted along the edges. Solve the equation $Ax = b$ "as well as possible" for the unknown potentials x_1 and x_2. Find a different set of batteries b_1, b_2, b_3 not all zero for which you can solve $Ax = b$ exactly.

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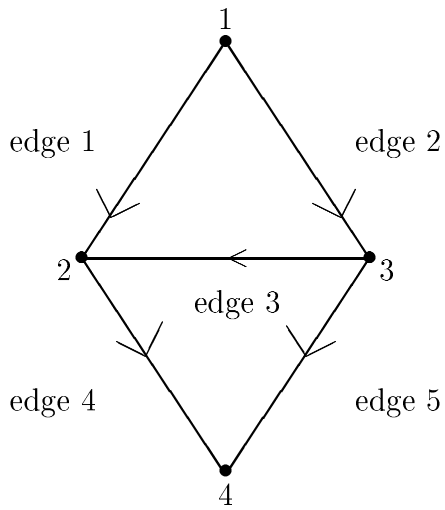
(c) Suppose you wanted to solve $A_0 x = b$ for all three unknowns x_1, x_2, x_3 (not grounded now). Why wouldn't the best $\bar{x}$ be computed right away from $A_0^{\mathrm{T}} A_0 \bar{x} = A_0^{\mathrm{T}} b$? How is the column space of A_0 (3 by 3) related to the column space of A (3 by 2)? What is the best $\bar{x}$?

(d) The **graph matrix** G (also called adjacency matrix) is n by n, if the graph has n nodes. Its entries are $G_{ij} = G_{ji} = 1$ if an edge connects nodes i and j, and otherwise $G_{ij} = 0$ (all $G_{ii} = 0$ on the diagonal). Write down G for the triangle graph and compute the matrix $A_0^{\mathrm{T}} A_0 + G$. Also add a fourth node below to the triangle graph, with edges from nodes 2 and 3, and compute $A_0^{\mathrm{T}} A_0$ and $A_0^{\mathrm{T}} A_0 + G$.



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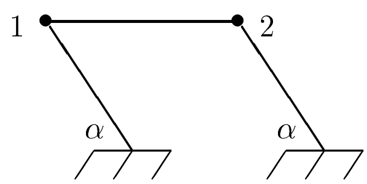
3) **(28 pts.)** **Truss Problem:** Consider the truss



where the slanting bars have equal length L, the top bar is horizontal, and the two bottom nodes are fixed in the ground (but bars can rotate). Further, let α denote the acute angle which the slanting bars make with the ground.

(a) On the basis of physical intuition, describe the infinitesimal displacements u of the free nodes which will produce no strain in the rods.

(b) Write down the matrix A which appears in the compatibility equation $e = Au$ for this truss. Use it to give a mathematical verification of the answer you gave in (a).

Note: Hanging springs and hanging rod in Exam 2! Best wishes.

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