18.075 - Practice exam 2 - Friday May 7, 2010 - 2.00pm

Your name:

No calculators, books, notes or cell phones may be used. Show all your work on these sheets: if you need extra space, you can write on the backs of pages.

1. (20 pts) Quick answers (no justification needed)

   - Legendre series representation of $3x^2$

   - $\int_{-\infty}^{+\infty} \sin(3x)\cos(4x)dx$

   - Orthogonality relation for eigenfunctions $y_n(x)$ of $y'' + \lambda e^{2x} y = 0$ on $0 \leq x \leq b$

   - Classify as linear, nonlinear or quasi-linear and state order of

     $$x(\partial_x \Phi)^2 + \partial_{yyy}\Phi = x\Phi$$

   - Classify as parabolic, hyperbolic or elliptic: $\partial_{xx}\Phi + \partial_{yy}\Phi + 2\partial_{xy}\Phi = x^2 y^2$
   - General solution of $x^2 y'' + xy' + (x^2 - 4)y = 0$

   - If $T(x, y) = X(x).Y(y)$ satisfies $\nabla^2 T = 0$, $x \in [0, 1], y \in [0, 1]$ with the boundary conditions $T(0, y) = 0$, $\partial_y T(x, 0) = 0$, $T(x, 1) = \partial_y T(x, 1)$ and $T(1, y) = 1$, do you first solve for $X(x)$ or for $Y(y)$?

   - Sketch the Fourier sine series representation of x^2, valid over $0 \leq x \leq 2$, over the domain $-4 \leq x \leq 4$

   - What is the weighting function in the following S-L problem ?

     $$y'' + x^3 y + \lambda \frac{1}{\cos(x)} y = 0, \qquad y(a) = y(b) = 0$$

2. (30pts) Solve the Laplace equation $\nabla^2 T(r,\theta) = 0$ outside the circle $r = R$ with the boundary conditions $T(R,\theta) = \cos^2(\theta)$.

3. (20pts) Use the Frobenius method to solve $xy'' + 2y' + xy = 0$.

4. (30pts) Find the solution $z(x, y)$ satisfying the PDE

$$\partial_{xx} z - 2\partial_{xy} z - 3\partial_{yy} z = 0$$

and the boundary conditions $z(x, y = 0) = x + 2$ and $z(x, y = x) = 2 - 16x^2$.