18.075 - Practice exam

Monday, March 15, 2010 - 2.00pm

No calculators, books, notes or cell phones may be used. Show all your work on these sheets: if you need extra space, you can write on the backs of pages. The total is 110 pts, which means that you can get 10 points of bonus if you solve everything.

1. (3'30) Evaluate the integral on the counterclockwise circle $\mathcal{C} : |z - i| = 3$

$$\oint_{\mathcal{C}} \frac{dz}{z(z+3)(z-2)}$$

2. (1'00) Find and classify the singularities in $\mathbb{C}$ of

- $$f(z) = \frac{\cos z}{z^2(z - \pi/2)}$$
- $$f(z) = \frac{3}{(z-2)(z+1)}$$

3. (10'00) Evaluate the integral on the contour $\mathcal{C} : z = 1 + iy, y : -\infty \to +\infty$

$$f(t) = \frac{1}{2\pi i} \int_{\mathcal{C}} \frac{2e^{zt} dz}{(z+1)(z^2+1)}$$

where $t > 0$. Write the solution $f(t)$ as a sum of real functions. Reminder: Draw the closed contour you are using and justify your solution. Hint: Use the left half.

4. (1'00) Calculate all possible values of i^i.

5. (6'00) Evaluate the real integral

$$\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$$

6. (5'00) Is $u(x, y) = x^3 - 3xy^2 + x^2 - y^2 - 2xy$ the real part of an analytic function? If so, find the imaginary part v of this function.

7. (4'00) Find

$$\int_{-\infty}^{+\infty} \frac{\sin(mx)dx}{x^2+4x+5}$$

for $m \geq 0$. At what stage in the argument have you used the fact that $m \geq 0$?

8. (4'00) Expand

$$\frac{z-1}{(z-2)(z-3)}$$

in Laurent series around $z = 2$. Find the annulus of convergence.

9. (10'00) Evaluate

$$\int_0^{+\infty} \frac{x^{1/4}}{x^2+x+1}$$